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DYNAMICS

FOR

BEGINNERS.

DYNAMICS

FOR

BEGINNERS

BY THE

REV. J.ohn B.ascombe LOCK, M.A.

SENIOR FELLOW, ASSISTANT TUTOR AND LECTURER
IN MATHEMATICS AND PHYSICS OF GONVILLE AND CAIUS COLLEGE;
LECTURER IN PHYSICS IN THE UNIVERSITY OF CAMBRIDGE;
FORMERLY MASTER AT ETON.

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PREFACE.

THIS work has been written in the hope of supplying a want which I believe is very widely felt; viz. of a book which explains the elementary principles of Dynamics, illustrating them by numerous easy *numerical* examples in a manner suitable for use in Schools with boys of ordinary mathematical attainments.

I have adopted a suggestion,—due I believe to Mr Hayward, F.R.S., of Harrow School—that the first part of the book should treat exclusively of Linear Dynamics; thus avoiding, at the beginning of the subject, all purely geometrical difficulties.

It will be found that, with the exception of one or two Articles, Sections I. and IV. (which may be read separately, and in which the fundamental Principles of Dynamics are explained) demand only a knowledge of Simple Equations in Algebra. In no case is any greater knowledge on the part of the Student assumed, than is denoted by Progressions in Algebra, the Trigonometry of one Angle, and (in Chapters IX. and X.) the Parabola.

I have ventured to suggest *names* for the units of velocity and acceleration, the use of which will be found to simplify considerably the language of the subject.

I should be greatly obliged to those who may make use of the book if they would point out any defects or obscurities in the text or would offer suggestions for its improvement.

J. B. LOCK.

GONVILLE AND CAIUS COLLEGE.
April, 1887.

NOTE.

The Examination Papers appended to the Book in most cases sufficiently indicate the range of the various Examinations.

The following suggestions however may be found useful.

For the Dynamics of the Additional Subjects in the Previous Examination at Cambridge, the Student may confine his attention to Section I., Section II. up to the end of Art. 63, and Art. 133 of Chapter XI.

For the First M.B. Examination, in addition to the above, the Student should read Section IV.

Those sections which are marked with an asterisk may in most cases be reserved with advantage for a second reading of the subject.

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EXAMINATION PAPERS.

SECTION I.

RECTILINEAR MOTION.

CHAPTER I.

VELOCITY, ACCELERATION.

1. The **distance** of a point P from a point A is represented in direction and magnitude by the straight line joining A to P.

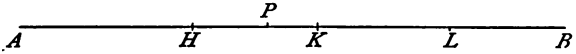
The point P is said to be *in motion relative to* the other point A, when the distance of P from A is changing.

The distance of P from A may be changing in *length*, or it may be changing in *direction*, or it may be changing *both in length and in direction.*

While the *direction* of the distance of P from A remains unchanged the path of P is the straight line AP.

We shall in this section discuss the motion of a point which moves always in a straight line.

Accordingly, for the present, we are not concerned with any change of direction in the distances considered.



Draw a straight line AB; let this line extend any length beyond A and beyond B. Let a point P be in motion relative to A in the line AB. Then the distance AP is different at different instants. Thus, suppose the point P at one instant to be at H; at another, at K; at another, at L; H, K and L being points *whose distances from* A *do not change*; then, P being in motion relative to A, P *alters its distance from* A *by the distance* HK in the course of a certain interval of time.

This is what is meant when it is said that P *passes over the distance* HK in a certain interval of time.

We choose **a foot** as our *unit distance.*

2. In what follows we shall frequently use the word **interval** as an abbreviation for *interval of time.*

We choose **a second** as our *unit interval.*

Let the distances from A of the above moving point P be observed at the ends of a series of equal intervals of time. Then if its motion is of such kind that in each of those equal intervals the point P passes over equal distances, and if this is true whatever be the length of these equal intervals, then we say that the point P is moving **uniformly**; hence,

3. DEFINITION. The motion of a point P relative to another point A is said to be **uniform** when it is such that the point P passes over equal distances in equal intervals of time, however short those intervals are.

Let us suppose that the point P is moving uniformly in the straight line AP. By its **velocity** we mean *something* which is doubled, trebled, etc. if the distance which it passes over in a given interval is doubled, trebled, etc., and which is halved, divided by three, etc. if the interval occupied in passing over a given distance is doubled, trebled, etc.; in other words,

4. DEF. The **velocity** of a point which is moving uniformly in a straight line is *that* which varies† directly as the distance passed over in a given interval, and which varies inversely as the interval occupied in passing over a given distance.

Velocity is therefore something which we observe in a moving point. It is capable of measurement; for we can say that the velocity of a point is double, or that it is half, or that it is some multiple, of the velocity of another point. We therefore choose a certain velocity as our *unit* velocity, and give it a name; just as we choose a unit of length and call it a foot; and a unit interval and call it a second...; hence

5. DEF. We choose as our **unit** *velocity* the velocity of a point which moving uniformly passes over the distance 1 foot in the interval 1 second.

---

† See Lock's Arithmetic, p. 162.

We shall call this *unit velocity* a **velo.**

A *velo* in a given direction, is *a foot per second* in that direction.

The word *per* here denotes *for every.*

NOTE. It is most important to notice that a point which has velocity requires an *interval of time* in which to pass over a *finite* distance.

A point P may have velocity *at* a certain instant, but unless the velocity continues during some interval after that instant, the point does not pass over any finite distance.

For example, a bullet which strikes a target is said to have a certain velocity *at* the instant at which it strikes.

Example i. A point which moves at the rate of 3 feet per second has a velocity 3 times as great as that of a point which moves at the rate of 1 foot per second. Hence its velocity is 3 *velos.*

Example ii. *Express in* velos *the velocity of a point which moves at the rate of* 60 miles per hour.

This point in 1 hour passes over 60 miles;

that is, in 60×60 seconds it passes over $60 \times 1760 \times 3$ feet;

but it passes over an equal distance in each second;

therefore in 1 second it passes over $\frac{60 \times 1760 \times 3}{60 \times 60}$ feet; that is, 88 feet.

Hence, its velocity is $\underline{\text{88 velos.}}$

Example iii. *A point has the* velocity 8 *velos; how many miles does it pass over in an hour?*

The point has 8 velos; therefore, in 1 second it passes over 8 feet; and it passes over equal distances in equal intervals;

therefore in 60×60 seconds it passes over $60 \times 60 \times 8$ feet;

that is, $\frac{8 \times 60 \times 60}{1760 \times 3}$ miles; or, $\underline{5\tfrac{5}{11}\text{ miles.}}$

EXAMPLES. I.

In each of the following questions the velocity given is uniform.

1. If I run 100 yards in 11 seconds, what is my velocity in velos?

2. A man runs at the rate of 7 velos; how long would he take to run a mile with this velocity?

3. Express in miles per hour (i) 40 velos, (ii) 100 yds. per minute.

4. How many velos has the extremity (i) of the minute hand of a clock which is 1 foot long, (ii) of the hour hand which is 10 in. long. [*The circumference of a circle*$=\frac{22}{7}$ *of the diameter.*]

5. How many velos has a man standing at the equator, in consequence of the daily revolution of the earth about its axis, the diameter of the earth being, say, 8000 miles?

6. The velocity of sound is 1118 velos; how long does it take to travel 13 miles?

7. Which is the greater velocity, 60 miles an hour or 500 yards in $11\frac{1}{4}$ seconds?

♠**8.** Find the ratio of the velocities $2\frac{1}{2}$ miles in 3 minutes and 10 ft. in a quarter of a second.

9. The following are the 'records' in foot racing (i) 100 yds. in 10 secs. (ii) a quarter of a mile in 49 secs. (iii) a mile in 4 min. $12\frac{3}{5}$ secs. (iv) ten miles in 51 min. $6\frac{3}{5}$ secs.; express the average velocity of each in velos.

10. The following are the records in bicycle racing (i) a mile in 2 min. 30 secs. (ii) 20 miles in 59 min. $6\frac{3}{5}$ secs. (iii) 22 miles 150 yds. in 1 hour; express the average velocity of each in velos and in miles per hour.

6. A point which has v velos passes over v feet in 1 second; it therefore passes over (t times v) feet in t seconds.

Hence, if s be the number of feet passed over in t seconds by a point which has a uniform velocity of v velos, we have

$$\mathbf{s=vt.}$$

Example. A point passes uniformly over a miles *in* b hours; *express its velocity in* velos.

In $b\times 60\times 60$ seconds it passes over $a\times 1760\times 3$ feet, therefore

$$\text{in 1 second it passes over } \frac{1760\times 3\times a}{60\times 60\times b} \text{ feet;}$$

$$\text{that is,} \qquad \frac{22a}{15b} \text{ feet;}$$

$$\text{therefore it has} \qquad \frac{22a}{15b} \text{ velos.}$$

7. The **measure** of the velocity of a point (when a velo is the unit) is the *number* of velos which it has.

This is the **ratio** of the number of feet passed over in any interval to the number of seconds in that interval.

For, with the notation of Art. 6, $s = vt$;

therefore $\mathbf{v} = \dfrac{\mathbf{s}}{\mathbf{t}}$.

Hence, *the velocity of a point* is *measured* by the **rate** at which it passes over distance per unit interval of time; in other words, by the **number** of units of length which it passes over in unit interval.

EXAMPLES. II.

The velocities given are uniform.

1. Express in velos a velocity of c miles per hour.

2. A point has v velos; how far does it go in h hours?

3. A point has k velos; how long does it take to go m yards?

4. A point goes m yards in t seconds; how many miles does it go in h hours supposing its velocity uniform?

5. A point goes λ inches in 1 minute; how far does it go in k days at the same rate?

6. A point goes k feet in t seconds; how long does it take to go m miles with the same velocity?

7. A train goes n miles in k hours; how far does it go in t seconds?

AVERAGE VELOCITY.

8. DEF. The **average velocity** of a moving point during a given interval, (in which it is not moving uniformly), is the velocity of another point which, *moving uniformly*, passes over the same distance in the same interval.

Example. A point moves over 6 ft., 7 ft., 8 ft. and 9 ft. *respectively, in four consecutive* seconds; *find its* average velocity *for the four seconds.*

In 4 seconds it passes over $(6+7+8+9)$ ft.; that is, 30 ft.

Thus, in 4 seconds a point having the required average velocity passes over 30 feet;

therefore in 1 second this point passes over $7\frac{1}{2}$ feet;

that is, the required average velocity is $\underline{7\frac{1}{2} \text{ velos}}$.

EXAMPLES. III.

1. A point passes over 9 ft., 10 ft., 11 ft. and 12 ft. in 4 consecutive seconds; find its average velocity for the 4 seconds.

2. Find the average velocity of the point in Question 1 (i) for the first 3 seconds, (ii) for the last 3 seconds.

3. A point passes over 20 yds., 24 yds., 28 yds., 32 yds. and 36 yds. in 5 consecutive seconds, shew that the average velocities for the 5 seconds, for the 3 middle seconds and for the 1 middle second are all equal.

4. A point has an average velocity of 2 velos, 3 velos, 4 velos, 5 velos, and 6 velos respectively in 5 consecutive intervals of 5 seconds each; find the average velocity for the 25 seconds; how far does the point go in the 25 secs.?

5. A point moves over a ft., $2a$ ft., $3a$ ft. and $4a$ ft. respectively in 4 consecutive intervals of b seconds each; find its average velocity for the whole time.

6. A point has an average velocity of $13v$ velos, $11v$ velos, $9v$ velos, $7v$ velos and $5v$ velos respectively in 5 consecutive intervals of t seconds each; find the average velocity for the $5t$ seconds.

7. A point has an average velocity of 3 velos for 2 secs., 6 velos for the next 4 secs., 10 velos for the next 4 secs. and 15 velos for the next 6 secs.; shew that in the 16 seconds it passes over the same distance as a point which has an average velocity of 3, 5, 7, 9, 11, 13, 15, 17 velos respectively in 8 consecutive intervals of 2 seconds each.

VELOCITY WHICH IS NOT UNIFORM.

9. Consider a point in motion not moving *uniformly*.

Its average velocity during any interval varies directly as the distance passed over in that interval and inversely as the length of the interval.

Its average velocity will be different for different intervals.

It is the *average* velocity of a train for a certain interval, which is found by noting the number of seconds occupied by the train in passing successive quarter-mile posts; but it is not the average velocity which is meant when we speak of the velocity of a point **at** a certain instant.

Consider two very long trains moving on parallel lines in the same direction. Let one of the trains (P) be moving with a constant velocity, say, 20 velos; let the other train (Q) start from rest and move gradually faster and faster. A passenger in train P will see his train gain on train

Q at first; but as time goes on, the train P goes faster, until at last there comes an instant at which the two trains are moving exactly together and neither is gaining on the other; this is only for an instant; it occurs *at* a certain instant. The train Q immediately begins to gain on the train P; but there is a particular instant *at* which neither train is gaining on the other and the two trains are *relatively at rest.* At that instant we say, that the velocity of the train Q is equal to the velocity of the train P; that is, the velocity of the train Q *at* that instant is 20 velos; hence,

10. DEF. The velocity of a moving point (which is not moving uniformly) **at** a given instant, is the velocity of another point which, moving with uniform velocity in the same direction, is at that instant at rest relatively to the first point.

UNIFORMLY INCREASING VELOCITY.

11. The simplest case of a point moving with velocity which is not uniform is that of a point whose velocity steadily increases or whose velocity steadily decreases.

When a point is said to have velocity which is increasing or decreasing, the point does not move for an interval, with uniform velocity and then suddenly move with a different velocity, but the velocity *at* any instant is the velocity which gradually increases or decreases.

12. DEF. A point is said to move with **uniformly increasing** velocity when in the course of each of a series of equal intervals its velocity receives an equal increment, no matter how large or how small these equal intervals may be.

The velocity *grows* steadily; it does not increase by jerks or steps.

Suppose that a point P is moving with uniformly increasing velocity; suppose that at a certain instant its velocity is 100 velos, and that after an interval of 1 second its velocity is 101 velos; then at the end of one more second its velocity will be 102 velos; at the end of another half-second its velocity will be $102\frac{1}{2}$; and so on. Its velocity increases *at the rate* of 1 velo per second.

Example. A point moving with uniformly increasing velocity has velocities 20 velos *and* 28 velos *respectively at the beginning and at the end of an interval of* 12 seconds, *find its velocity* 3 seconds *before the beginning of the interval and* 3 seconds *after the end of the interval.*

In 12 seconds the velocity increases by 8 velos; therefore

in 1 second the velocity increases by $\frac{8}{12}$, or $\frac{2}{3}$ velo.

Hence, 3 seconds before the instant at which it has 20 velos it has $(20 - 3 \times \frac{2}{3})$ velos, or 18 velos. And 3 seconds after the instant at which it has 28 velos, it has $(28 + 3 \times \frac{2}{3})$ velos, or 30 velos.

13. A similar definition applies to the expression **uniformly decreasing** velocity.

Example. A point moving with uniformly decreasing *velocity has* 30 velos; *after* 5 seconds *it has* 20 velos; *when will it come to rest?*

The velocity diminishes by 10 velos in 5 seconds,

therefore it diminishes by 30 velos in 15 seconds,

When it has diminished by 30 velos it has 0 velo and is at rest. Therefore it comes to rest in 15 secs.

EXAMPLES. IV.

In the following examples the motion is in each case that of uniformly increasing or decreasing velocity.

1. A point has 8 velos at the beginning and 9 velos at the end of a certain second; how many velos has it after 4 more seconds?

2. In a certain interval of half a minute the velocity of a point increases from 10 velos to 100 velos; what was its velocity at the middle of the interval?

3. In 1 hour the velocity of a point decreases from 300 velos to 120 velos; what was its velocity at the end of each quarter of that hour?

4. At noon a point is moving with 20 velos; at 4 P.M. it has 100 velos; what velocity has it at 2.15 P.M.?

5. At 2 o'clock a point has 7 velos; at 2.45 it has 142 velos; what has it at 3.30?

6. A train is being pulled up, and moves with uniformly decreasing velocity; at a certain time it is going 60 miles per hour, after 18 seconds it has 80 velos; when will it stop?

7. A point which started from rest has after 4 seconds a velocity of 100 yds. per minute; when will it have 10 velos? and when will it be going at the rate of 60 miles an hour?

8. A point at a certain time has 11 velos; 2 minutes later it is moving with a velocity of 30 miles an hour; when was it at rest?

9. A point has 128 velos and its velocity is decreasing at the rate of 32 velos per second; when will it come to rest? and how long will it be before it again has 128 velos?

10. A point moving at the rate of 11 ft. per second has its velocity increased by 7 ft. per second per second; when will it be moving at the rate of 60 miles an hour?

11. A point whose velocity is decreasing at the rate of (32 ft. per second) per second comes to rest after 3 seconds; what was its velocity at the beginning of the 3 seconds? and what at the middle of the 3 seconds? and what was it after 3 seconds more?

12. A point whose velocity is increasing at the rate of 32 ft. per second per second is moving vertically downwards with a velocity of 112 ft. per second; when was it at rest? and when was it moving vertically upwards with a velocity of 112 ft. per second?

14. When a point is moving with uniformly increasing velocity, its *average* velocity in a given interval of time is greater than the velocity which the point has at the *beginning* of the interval and is less than the velocity which the point has at the *end* of the interval.

For its velocity at the beginning of the interval is less than its velocity at any other instant in the interval, and its velocity at the end of the interval is greater than its velocity at any other instant in the interval.

15. The same thing may also be stated thus;

Let a point, moving with uniformly increasing velocity, at the beginning of a certain interval of t seconds have u velos, and let the point at the end of the interval have v velos; then, the distance passed over by the point in this particular interval of t seconds is greater than $t \times u$ feet and is less than $t \times v$ feet.

Example i. *A point has* 8 velos *and its velocity is increasing* at the rate of 2 velos per second; *shew that in the next* four seconds *it passes over more than* 44 feet *and less than* 52 feet.

Its velocity at the *beginning* of each second is 8, 10, 12, 14 velos respectively; therefore it passes over *more* than $(8+10+12+14)$ feet in these 4 seconds.

Its velocity at the *end* of each second is 10, 12, 14, 16 velos respectively; therefore it passes over *less* than $(10+12+14+16)$ feet in these 4 seconds.

Example ii. *Shew by taking the velocity of the point in* Example i *at the beginning and at the end of each* quarter second, *that the distance passed over in the* 4 seconds *lies between* 47 feet *and* 49 feet.

At the *beginning* of each $\frac{1}{4}$ second the velocities are

$8, 8\frac{1}{2}, 9, 9\frac{1}{2}, 10, 10\frac{1}{2}$ etc.…up to $15\frac{1}{2}$, velos, respectively,

and the distance passed over is therefore *greater* than

$$\tfrac{1}{4}\{8+8\tfrac{1}{2}+9+9\tfrac{1}{2}+\ldots\ldots\ldots\ldots+15\tfrac{1}{2}\} \text{ feet,}$$

that is, greater than 47 feet.

At the *end* of each $\frac{1}{4}$ second the velocities are

$8\frac{1}{2}, 9, 9\frac{1}{2}$ etc.…up to 16, velos, respectively,

and the distance passed over is therefore *less* than

$$\tfrac{1}{4}\{8\tfrac{1}{2}+9+9\tfrac{1}{2}+\ldots\ldots\ldots\ldots+16\} \text{ feet,}$$

that is, *less* than 49 feet. Q. E. D.

EXAMPLES. V.

The following velocities are each uniformly increasing or decreasing.

1. A point has 8 velos and its velocity is increasing uniformly at the rate of 1 velo per second; shew by considering the velocity at the beginning and at the end

(i) of each second, that the distance passed over in the next 10 seconds is greater than 125 ft. and less than 135 ft.

(ii) of each $\frac{1}{10}$th of a second, that the distance passed over in the 10 seconds is greater than $129\frac{1}{2}$ ft. and less than $130\frac{1}{2}$ ft.

2. A point has 120 velos and its velocity is decreasing at the rate of 32 velos per second; shew by considering the velocity at the end of each $\frac{1}{32}$th of a second that the distance passed over in the next second is less than $104\frac{1}{2}$ ft. and greater than $103\frac{1}{2}$ ft.

3. A point has 16 velos and its velocity is increasing at the rate of 32 velos per second; shew by dividing the interval of time into $\frac{1}{32}$ths of a second that the distance passed over in the next 3 seconds cannot differ from 192 feet by more than 18 inches.

4. A point is at rest and velocity is given to it uniformly so that at the end of 1 second it has 32 velos; shew as in example **3**, that in the 3rd second of its motion the distance which it passes over does not differ from 80 feet by more than 6 inches.

5. A point has u velos, and its velocity is increasing at the rate of 32 velos per second; shew, by dividing the next 8 seconds into $8n$ equal parts, that the distance passed over in those 8 seconds cannot differ from $(8u+1024)$ feet by more than $\frac{128}{n}$ feet.

6. A point has u velos, and its velocity is increasing at the rate of a velos per second; shew, by dividing the next t seconds into n equal parts, and considering the velocity at the beginning and the end of these n intervals, that the distance passed over in t seconds cannot differ from $(u+\frac{1}{2}at)\times t$ feet by more than $\frac{at^2}{2n}$ feet.

16. It is of the greatest importance that the student should mentally realize the motion of a point moving with uniformly increasing (or decreasing) velocity.

Consider again the case of two very long trains A and B moving on parallel lines in the same direction; suppose them to be passing a certain station H,—the train A with a constant velocity 20 velos,—the train B with a constantly increasing velocity which, when the engine is at H, is 4 velos.

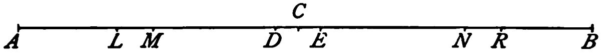
At H suppose the engines are abreast; at first A will gain upon B; but it will gradually gain more and more slowly, until an instant arrives when the two trains are relatively at rest; after which time B will at once begin to gain upon A; and will gradually gain at a faster and faster rate.

Hence an instant arrives when the engines are again abreast; suppose this to occur at a place K; then each train has passed over the distance from H to K in the same interval; hence the average velocity of B *for the time occupied in going from H to K* is the velocity of A, viz. 20 velos.

We proceed to prove that *the instant at which* the two trains are relatively at rest, *is at the middle of the interval of time* which they each take in going from H to K.

The proof given on the next page may be deferred if it is thought desirable. An alternative proof is given in Art. 24.

17. PROP. *When a point is moving with uniformly increasing velocity, its* **average velocity** *for any given interval of time is equal to the velocity which the moving point has* **at the middle of that interval of time.**



Let the given interval of time t seconds be represented by the length of the finite straight line AB.

Divide the t seconds into any *odd* number n of smaller equal intervals, each containing τ seconds.

Let DE represent the *middle* interval of τ seconds; and let C be the middle instant of AB, and therefore of DE. Take a pair of the little intervals LM and NR, one on each side of C and equidistant from C.

Let the moving point *at* the instant D have v' velos; *at* the instant C, v velos; and at the instant E, v'' velos; let w be the number of velos added on in the interval LD.

At the instant L the moving point has $(v' - w)$ velos; at the instant N it has $(v' + w)$ velos; at the instant M it has $(v'' - w)$ velos; at the instant R it has $(v'' + w)$ velos. [Since the number of velos which it has increases uniformly with the time, and $LD = ME = DN = ER$.]

The distance passed over by the point in the intervals LM and NR together, is therefore greater than $\{(v' - w)\tau + (v' + w)\tau\}$ feet, and less than $\{(v'' - w)\tau + (v'' + w)\}\tau$ feet; that is, greater than $2\tau v'$ feet, and less than $2\tau v''$ feet.

Also, in the middle interval DE, the distance passed over is greater than $\tau v'$ ft. and less than $\tau v''$ ft.

Hence, the whole distance passed over in all the n intervals is greater than $n\tau \times v'$ ft. and less than $n\tau \times v''$ ft.; that is, greater than tv' feet and less than tv'' feet.

This is true however great the odd number n may be.

Let n be increased without limit; then τ is diminished without limit and v' and v'' each approach v as their limit. Therefore the distance passed over in the whole interval t seconds (which distance is *between* tv' feet and tv'' feet) *cannot be other than* tv feet.

In other words, its average velocity for the time t seconds is v velos; and this is the velocity which the point has at C, the middle of that interval of t seconds. Q. E. D.

Example i. *A point has* 18 velos, *and is moving with uniformly increasing velocity, so that* 1 second *later it has* 20 velos; *how far does it go in that second?*

The average velocity for the 1 sec. is the same as the velocity which the point has at the middle of the interval, that is, after half a second.

Since in 1 second the velocity is increased by 2 velos,
therefore in ½ second the velocity is increased by 1 velo;
hence the average velocity for the 1 second is (18 + 1), or, 19 velos.

In 1 second it passes over 19 × 1 ft., or 19 feet.

Example ii. *How far does the point in* Example i. *go in* 8 seconds?

The middle of the interval of 8 seconds is 4 seconds after the time when it has 18 velos.

In 1 second the velocity is increased by 2 velos,
therefore in 4 seconds the velocity is increased by 8 velos;
hence, the average velocity for 8 seconds is (18 + 8) or 26 velos;
therefore, the distance passed over is 26 × 8 ft.; that is, 208 feet.

Example iii. *A point, moving with uniformly decreasing* velocity, *passes over* 294 ft. *in* 7 secs.; *and at the beginning of the* 7 seconds *has* 49 velos; *find its* velocity *after* 3 seconds *more.*

Since it passes over 294 ft. in 7 seconds, its average velocity [Art. 8] is 42 velos;
its actual velocity at the middle of the 7 secs. is equal to this average velocity;
hence, since at the beginning of the 7 seconds it has 49 velos, and 3½ seconds later it has 42 velos, therefore its velocity decreases by 7 velos in 3½ secs.; or, by 2 velos per second;

hence, after 10 seconds it will have (49 − 20) or 29 velos.

EXAMPLES. VI.

In the following examples the velocity is uniformly increasing (or decreasing).

1. A point has 100 velos at 12 o'clock, and its velocity increases to 104 in 1 second; find

(i) how far it goes in 1 second,
(ii) how far it goes in 2 seconds,
(iii) how far it goes in the 30 seconds after 12.1′ o'clock,
(iv) how far it goes between 12.1′ and 12.2′ o'clock.

2. At a certain time a point has 16 velos, and its velocity increases at the rate of 32 velos per second; shew that in the next three seconds it passes over 192 ft. [See Ex. V. 3.]

3. A point has 120 velos, and its velocity is decreasing at the rate of 32 velos per second; shew that in the next second it passes over 104 ft. [See Ex. V. 2.]

4. A point in a certain interval of 3 seconds passes over 300 feet; in the next 4 seconds it passes over 764 ft.; shew that its velocity is increasing at the rate of 26 velos per second.

5. A point in a certain interval of 5 seconds passes over 125 feet; in the next 8 seconds it passes over 304 feet; when did it start from rest?

6. A point is observed to pass over 120 ft., 129 ft., 138 ft. respectively in 3 consecutive seconds; is this consistent with uniformly increasing velocity?

7. Starting from rest, in the 5th second of its motion, a point passes over 144 ft.; find its velocity after 10 seconds from rest.

8. A point passes over 144 ft. in a certain interval of 2 seconds and comes to rest after 3 seconds more; how much further did it move?

9. A point starts from rest, and after 10 seconds it has 25 velos; how far does it go in the 20th second of its motion?

10. A point passes over 24 ft. in a certain second; at the end of that second it has 28 velos; how far does it go in the next second? and when is it at rest?

11. A point passes over 144 ft. in two seconds and over 96 ft. in the next two seconds; when will it come to rest? and how far does it move before doing so?

12. A train goes 10 miles in a quarter of an hour, having started from rest; shew that if its velocity has been uniformly increasing, it is moving at the end of that quarter of an hour with a speed of 80 miles per hour.

ACCELERATION.

18. DEF. When a point is moving with uniformly increasing velocity, its **acceleration** is *that* which varies directly as the *increase* of velocity in a given interval, and inversely as the interval required for a given increase of velocity. [Compare Arts. 3 and 4.]

Thus, the acceleration of a point is doubled, or trebled etc. when the velocity added in a given interval is doubled, or trebled etc.; and it is halved, or divided by three, etc. when the interval (in which a given velocity is added) is doubled, or trebled etc.

19. We shall choose as our **unit** *acceleration* the acceleration of a point which, moving with uniformly increasing velocity, has its velocity increased by 1 velo in the course of each second.

We shall call this *unit acceleration* a **celo.**

Thus, *a celo* is (*a foot per second*) *per second.*

NOTE. It is most important to notice that a point which has acceleration requires an interval of time in which to increase its velocity by any definite amount.

Example i. A point which has 6 velos added to its velocity in the course of each second, has six times the acceleration of a point having the unit acceleration; hence it has 6 *celos.* The same point has 1 velo added in the course of the $\frac{1}{6}$th part of a second.

Example ii. *A point has at a certain instant* 12 velos *and it has* 6 celos; *how many* velos *will it have* 10 seconds *later?*

In the course of 1 second 6 velos are added,
therefore in 10 seconds 60 velos are added.
So that after 10 secs. the point has $(12+60)$ velos, or 72 velos.

Example iii. *How far does the point in* Ex. ii *go in those* 10 secs.

The average velocity of the point in the 10 seconds is that which it has at the middle of the interval; that is, $(12+5\times 6)$ velos, or 42 velos; therefore, in 10 seconds the point goes 42×10 ft., or 420 feet.

Example iv. *A point having* 32 celos *starts* from rest; *how far will it go in the* 4th second *of its motion? and how far in the* 12th second?

At the middle of the 4th second, that is, after $3\frac{1}{2}$ secs., it has
$3\frac{1}{2} \times 32$ velos, or 112 velos; hence,
during the 4th second its average velocity is 112 velos;
therefore, in that second it passes over 112×1 ft., or $\underline{112 \text{ ft.}}$

Again, at the middle of the 12th second; that is, at the end of $11\frac{1}{2}$ secs. it has $11\frac{1}{2} \times 32$ velos, or 378 velos; therefore,
in the 12th second its average velocity is 378 velos;
so that in that second it passes over $\underline{378 \text{ ft.}}$

Example v. *A point which has* u velos, *has* u' velos *added on in* t seconds; *what is its* acceleration?

When 1 velo is added in 1 second the point has 1 celo;
therefore, when 1 velo is added in t seconds the point has $\frac{1}{t}$ celos,
and when u' velos are added in t seconds the point has $\frac{u'}{t}$ celos.
Therefore the required acceleration is $\frac{u'}{t}$ celos. Hence

20. The *measure* of the acceleration of a point is the **ratio** of the *number* of velos added in an interval to the *number* of seconds in that interval.

In other words, the acceleration of a point is *measured* by the **rate** at which its velocity increases per unit-interval of time

or, by the **number** of units of velocity added on per unit-interval.

21. When the velocity of a point is *decreasing* uniformly it is said to have **negative acceleration**.

When the acceleration of a point is opposite in direction to its velocity it is often called **retardation**.

Suppose a point to have 32 velos and 1 second after to have 30 velos, we say it has -2 celos. This is consistent with the actual result and is a very convenient way of speaking.

This point 5 seconds later will have $(32 - 2 \times 5)$ velos.

EXAMPLES. VII.

In these examples the motion in each case is uniformly accelerated in one straight line.

1. A point has 16 velos and 32 celos, find

(i) its velocity after 2 seconds,
(ii) its velocity after 5 seconds,
(iii) how far it goes in the first 4 seconds,
(iv) how far it goes in the first 20 seconds,
(v) when it will have 1100 velos,
(vi) when it was at rest.

2. A point has 200 velos; 5 seconds later it has 300 velos; find its acceleration.

3. A point passes over 144 ft. in 3 seconds and has 32 celos; shew that it started from rest.

4. A point passes over 144 ft. in 3 seconds and has 32 celos; how far will it go in the next 4 seconds?

5. In a certain interval of 5 seconds a point passes over 250 ft. and in the next 8 seconds it passes over 608 ft.; find its acceleration.

6. A point starts from rest, and in the 10th second of its motion it passes over 304 ft.; find its acceleration and the distance it passes over in the 5th second of its motion.

7. A point has 30 velos and -5 celos; find

(i) its velocity after 3 seconds,
(ii) when and where it comes to rest,
(iii) the distance passed over in 5 seconds,
(iv) the ratio of the distance passed over in the first 3 seconds to that passed over in the next 3 seconds,
(v) when it has gone half of the distance which it goes in the first 6 seconds.

8. In a certain interval of 3 secs. a point passes over 288 ft.; in next 2 secs. it passes over 32 ft.; find

(i) its acceleration,
(ii) when it is at rest,
(iii) where it is after 3 more seconds,
(iv) when it has 144 velos,
(v) when it has -144 velos.

9. A point passes over h feet in t seconds; in the next $2t$ seconds it passes over k feet; find its acceleration.

10. A point passes over h feet in 2 seconds; after an interval of t seconds it is observed to pass over k feet in 2 seconds; find its acceleration.

22. PROP. *Let a point have* u velos *at the beginning of a certain interval of* t seconds; *let it have* a celos; *and let* s feet *be the distance it passes over in the* t seconds; *then*

$$\mathbf{s=ut+\tfrac{1}{2}at^2} \qquad \text{(i).}$$

For, the *average velocity* of the point for the t seconds is that which it has *at* the *middle* of the interval; that is, the velocity which it has after $\frac{1}{2}t$ seconds.

But a velos are added to its velocity per second; therefore, ($\frac{1}{2}t$ times a) velos are added on in $\frac{1}{2}t$ seconds; so that its average velocity for the t seconds is $(u+\frac{1}{2}at)$ velos.

Therefore, the distance passed over in the t seconds is

$$[(u+\tfrac{1}{2}at)\times t] \text{ feet;}$$

hence, $$s=ut+\tfrac{1}{2}at^2. \qquad \text{Q. E. D.}$$

23. In future, when using the letters s, u, a or t we shall always suppose them to have the meaning given them in Art. 22.

24. The results of Arts. 17 and 22 may also be proved by the method of Examples V. **6** as follows:

To prove, with the usual notation, that

$$s=ut+\tfrac{1}{2}at^2.$$

Let the interval t seconds be divided into n equal intervals each of τ seconds; so that, $n\tau=t$.

The velocities at the beginning of these n intervals are respectively	The velocities at the end of these n intervals are respectively
u velos,	$(u+a\tau)$ velos,
$(u+a\tau)$ velos,	$(u+2a\tau)$ velos,
$(u+2a\tau)$ velos,	$(u+3a\tau)$ velos,
...	...
...	...
$(u+\overline{n-1}\,a\tau)$ velos.	$(u+na\tau)$ velos.

Therefore the sum of the distances passed over by the moving point in these n intervals is [Art. 14] *greater* than

$$[u\tau + (u + a\tau)\,\tau + (u + 2a\tau)\,\tau + \ldots + (u + \overline{n - 1}\,a\tau)\,\tau] \text{ feet,}$$

and is *less* than

$$[(u + a\tau)\,\tau + (u + 2a\tau)\,\tau + \ldots + (u + na\tau)\,\tau] \text{ feet.}$$

That is, the required number of feet s is greater than

$$\tfrac{1}{2}n\,\{2u\tau + (n - 1)\,a\tau^2\}$$

and less than $\tfrac{1}{2}n\,\{2\,(u + a\tau)\,\tau + (n - 1)\,a\tau^2\}$; that is,

s is greater than $\left\{un\tau + \tfrac{1}{2}\,an^2\tau^2 - \tfrac{1}{2}\,\dfrac{an^2\tau^2}{n}\right\}$,

and s is less than $\left\{un\tau + \tfrac{1}{2}\,an^2\tau^2 + \tfrac{1}{2}\,\dfrac{an^2\tau^2}{n}\right\}$;

but $n\tau$ is t, so that

s is greater than $ut + \tfrac{1}{2}at^2 - \tfrac{1}{2}\,\dfrac{at^2}{n}$,

and s is less than $ut + \tfrac{1}{2}at^2 + \tfrac{1}{2}\,\dfrac{at^2}{n}$.

This is true however great n may be; and $\tfrac{1}{2}\,\dfrac{at^2}{n}$ diminishes as n increases.

Let n be increased without limit; then s lies *between* two numbers each of which differs from $ut + \tfrac{1}{2}at^2$ by a number $\left(\tfrac{1}{2}\,\dfrac{at^2}{n}\right)$ which is diminished without limit.

Therefore s cannot be different from $ut + \tfrac{1}{2}at^2$. Q. E. D.

25. Let v velos be the velocity of the point at the end of the interval of t seconds, then

$$v \text{ velos} = (u + at) \text{ velos}$$

or, $$v = u + at \qquad \text{(ii)},$$

but, $$s = ut + \tfrac{1}{2}at^2,$$

therefore, $$2as = 2aut + a^2t^2,$$

hence, $$\begin{aligned} 2as &= u^2 + 2uat + a^2t^2 - u^2 \\ &= (u + at)^2 - u^2 \\ &= v^2 - u^2 \qquad \text{(iii).} \end{aligned}$$

26. These three results are of great importance; viz.

$$\mathbf{v}=\mathbf{u}+\boldsymbol{a}\mathbf{t} \qquad \text{(i),}$$

$$\mathbf{s}=\mathbf{u}\mathbf{t}+\tfrac{1}{2}\boldsymbol{a}\mathbf{t}^2 \qquad \text{(ii),}$$

$$\boldsymbol{a}\mathbf{s}=\tfrac{1}{2}\mathbf{v}^2-\tfrac{1}{2}\mathbf{u}^2 \qquad \text{(iv),}$$

it being understood, that, in an interval of t seconds, *a point which has* a celos *has* u velos *at the beginning, and* v velos *at the end of the* t seconds; *and during the* t seconds *passes over* s feet.

Example i. How long *will a point which has initially* 400 velos *and* -32 celos, *take to go a distance of* 1600 feet *from its initial position?*

Let the required interval be x seconds; then since

$$s=ut+\tfrac{1}{2}at^2,$$

therefore, $1600=400x-16x^2,$

or, $x^2-25x+100=0,$

or, $(x-20)(x-5)=0.$

So that, either $x=20$, or $x=5$.

Thus the required interval is either *5 seconds*, or *20 seconds*.

The point has -32 celos; that is, its velocity is decreasing. In 5 seconds it passes over 1600 ft.; then it goes on, comes to rest, and returns to the same point, after 15 seconds more.

Example ii. *A point has initially* 20 velos *and it has* 5 celos; how far *will it go in attaining a velocity of* 100 velos?

Let the required distance be x feet; then since

$$as=\tfrac{1}{2}v^2-\tfrac{1}{2}u^2,$$

therefore, $5x=\tfrac{1}{2}(100^2-20^2),$

or, $5x=5000-200.$

that is, $x=960.$

The required distance is *960 feet.*

Example iii. *A point is moving with a* velocity *of* 15 miles per hour, *and has an* acceleration *of* 1 mile per minute per minute; *how far does it go in the next minute?*

15 miles per hour = 22 velos. [Ex. ii, p. 3.]

By an acceleration of 1 mile per minute per minute, is meant that an additional velocity of one mile per minute is produced in each minute.

Now, a mile per minute $=\frac{1760}{60}\times 3$ ft. per second,

that is, 88 velos.

Hence, 88 velos are added on in the course of each minute;

that is, $\frac{88}{60}$ velos are added on per second.

Hence, 1 mile per min. per min. = $\frac{88}{60}$ celos.

Therefore the point will move over

$$\left\{22 \times 60 + \tfrac{1}{2} \cdot \tfrac{88}{60} \times (60)^2\right\} \text{ feet in } 60 \text{ secs.};$$

that is, $(1320 + 2640)$ ft., or, $\underline{3960 \text{ ft.}}$

EXAMPLES. VIII.

The motion in each case is that of uniform acceleration in a straight line.

1. A point has initially 96 velos and it has -32 celos; in how many seconds will it be at a distance 144 ft. from its initial position?

2. A point has initially 20 velos, and it has 2 celos; how long will it take to go 100 ft.? and how far will it go in attaining a velocity of 25 velos?

3. One point P has initially 200 velos and it has 32 celos; a second point Q has initially 1000 velos and 16 celos; initially they start from the same point in the same direction; when are they again together? and how far will they have gone?

4. A point starts from rest with 32 celos, and from the same place, at the same time and in the same direction another point starts with initially 40 velos and 30 celos; when and where will the first point overtake the second? and when will they have equal velocities?

5. A point having initially 20 velos attains a velocity of 30 miles per hour in passing over 128 ft.; find its acceleration.

6. A point after passing over a quarter of a mile with an acceleration of 20 ft. per second per second, attains a velocity of 2704 ft. per second; what was its initial velocity?

7. A train in passing over a quarter of a mile increases its velocity from 5 miles an hour to 35 miles an hour; what is its acceleration?

8. A train moving at the rate of 45 miles an hour is stopped by the continuous brake in 220 yds.; what is its 'acceleration'?

9. A train has a velocity of 60 miles per hour and it has a retardation of one mile per minute per minute; how far will it go before it comes to rest?

10. A point in passing over a mile has its velocity increased from a mile in 3 minutes to a mile in $2\frac{1}{2}$ minutes: how many celos has it?

11. A point passes over 100 yards in 10 seconds, and the next 100 yards in 12 seconds; when will it be 100 yards further on?

12. A point passes over 200 ft. in 12 seconds, and the next 300 ft. in 20 seconds; when will it be 400 ft. further on?

MISCELLANEOUS EXAMPLES. IX.

The motion in each case is uniform acceleration in a straight line.

1. A point moves over 7 ft., 9 ft., 11 ft., 13 ft. respectively in four consecutive seconds; find its average velocity; and supposing the velocity to be uniformly increasing, find its acceleration and its velocity at the beginning and at the end of the four seconds.

2. A point has 4 celos for 10 seconds and passes over 1000 ft. in those 10 seconds; find its initial velocity; and if it had 4 celos from rest, find when and where it started from rest.

3. At the beginning of an interval of 12 seconds a point has 40 velos, and its average velocity for the 12 seconds is 80 velos; find its acceleration.

4. A point passes over 5 ft. in one second, 16 ft. in the next 2 secs., 11 ft. in the next second, 45 ft. in the next 3 seconds; shew that this is consistent with a constant acceleration, and find the acceleration.

5. A point has initially 20 velos and it has -2 celos; find when it will be 100 ft. from the initial position.

6. A point has initially 20 velos and it has 32 celos; find how long it will take to go 1800 feet.

7. A point has initially 120 velos and it has -32 celos; find when it will be 200 ft. from the starting point.

8. A point has 20 velos and it goes twice as far in the third and fourth seconds together as it does in the first two seconds of its motion; find the acceleration.

9. A point starts from rest with a constant acceleration; shew that it goes in the second half of any interval three times as far as it does in the first half.

10. Two points move from the same point in the same line, one with a constant velocity 4 velos, the other from rest with 2 celos; they start together; when will they be together again?

11. A point P starts along a line with constant acceleration a celos, and t seconds later a point Q starts along the same line with constant velocity u velos; prove that Q will not overtake P unless u is greater than at.

12. The line AB is 192 ft. long; P starts from A along AB with 8 velos and moves uniformly; Q starts from rest at B along BA with 8 celos; find where they will meet.

13. Two points P and Q have each 32 celos in the same direction AB; P starts from A from rest and Q starts simultaneously from B having a velocity towards B just sufficient to carry it to B; find where they pass each other, if AB is 120 ft.

14. Two points P and Q have each g celos in the same direction AB; P starts from A from rest and Q starts simultaneously from B (which is a ft. from A) having a velocity of $\sqrt{(2ag)}$ velos towards B; find where they pass each other.

15. Two points P and Q have each g celos in the same direction, say vertically downwards; P starts from rest from a point A and Q starts from B, vertically below A, with a certain velocity v celos upwards. Shew that however small v may be, P will always overtake Q.

16. A point moves from rest over a certain distance with 32 celos; it describes $\frac{16}{25}$ of the whole distance during the last second of the motion; find the whole interval occupied.

17. A point has initially mn velos and $-n$ celos; shew that after $(n+k)$ seconds it returns to the same point which it was passing at the end of the first $(n-k)$ seconds, with the same number of velos.

18. Prove that the distances passed over in successive equal intervals by a point starting from rest and moving with a constant acceleration are proportional to the series 1, 3, 5, 7...

19. A point having a certain velocity, moves with an acceleration in the opposite direction, until it comes to rest; prove that the intervals of describing the first half and second half of its path respectively are as $1 : \sqrt{2}+1$.

20. A point having a celos, passes over h feet in a certain interval, with an average velocity u velos, and its velocity is increased during the interval by v velos; prove that $uv=ah$.

21. An engine-driver, whose train is travelling at the rate of 30 miles an hour, sees a danger signal at the distance of 220 yards, and does his best to stop the train; supposing that he can stop the train when travelling 30 miles an hour in 440 yards, shew that his train will reach the danger signal with a velocity of $21\frac{1}{5}$ miles per hour nearly.

22. A point, moving with a constant acceleration a celos, passed over twice as many feet in a certain interval t seconds as it did in the immediately preceding interval of t seconds; shew that its velocity at the beginning of the first interval was $\frac{1}{2}ta$ velos.

CHAPTER II.

MASS AND FORCE.

27. Consider some definite quantity of matter; say, a cubical lump of iron. This (i) consists of a certain *kind of material* (iron), (ii) has a definite *shape*, (iii) has a definite *volume*, (iv) contains a certain *quantity of matter*, (v) has a definite *weight*.

Each of these (i), (ii), (iii), (iv), (v) is a separate and distinct idea applicable to the lump of iron.

In dynamics we are chiefly concerned with the last two; viz., the *quantity of matter*, which we shall call the **mass**, and the *weight*.

We proceed to consider what is meant by mass.

28. We may illustrate the idea of *mass* as follows:

Let us imagine a perfectly smooth, perfectly horizontal sheet of ice†. Upon this, place a lump of matter, such as a smooth block of stone in the form of a cubic foot. Now urge the lump forward with a horizontal pressure or force, taking care to apply the force constantly and uniformly for an interval of, say, 10 seconds. What happens? It will be found that by the force the lump has a certain amount of acceleration given to it; so that a velocity grows in it *as long as the force is applied;* and at the end of the 10 seconds it will have acquired a certain number of velos. At the end of the 10 seconds let the force be withdrawn. What happens? The lump has a certain number of velos; and it will be found that it will continue to move uniformly with that number of velos, so long as there is nothing in the nature of a force applied to it.

29. Suppose now that we try the experiment of Art. 28 with *several* lumps of the same size but of different material; say, one of lead, one of stone, one of cork; and suppose that we can apply an equal constant horizontal force to each continuously for an interval of, say, 10 seconds. What happens? It will be found that the lead, the stone, and the cork, have each a constant number of celos communi-

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† It must be noticed that these conditions can only be approximately fulfilled; the best ice is not perfectly smooth and the movement of the air would interfere to a certain degree with such an experiment.

cated by the force; but that the number of celos given to the cork is greater than the number given to the stone, and the number given to the stone greater than the number given to the lead. So that at the end of the 10 seconds they will each have acquired a certain number of velos, the cork more than the stone, the stone more than the lead[†]; also, that after the force has been withdrawn, each will continue to move with its own constant velocity, so long as there is nothing in the nature of a force applied to it.

30. DEF. We shall choose as our **unit** *mass* the mass of a certain piece of metal [called 1 lb. (avoirdupois).]

We shall call this *unit mass* a **pound**, or, 1 **lb.**

31. DEF. **Force** is that which when applied to *mass* produces, or tends to produce, in it acceleration in the direction of the force; so that the force varies as the acceleration produced in a given mass; and also varies as the mass in which a given acceleration is produced.

Consider (i) the experiment of Art. 27; here, if we double the force, we shall find that we produce in the same mass double the acceleration; consider (ii) the experiment of Art. 28; here, if the stone has 5 times the '*mass*' of the cork, we must have 5 times the force acting on the stone to produce in it the same number of celos as in the cork.

32. DEF. We shall choose as our **unit** *force* that force which acting on *a pound* produces in it 1 celo.

We shall call this *unit force* a **poundal.**

33. The statement of Art. 31 is to be given the fullest possible interpretation; it asserts that

I. *Force* is *that* which produces *acceleration* in *mass*; therefore, whenever a mass has acceleration, it is under the action of some external force.

II. When *no external force* acts on a mass it has *no acceleration*; in other words, if at rest, the mass remains at rest, and if in motion, the mass continues to move with uniform velocity.

[†] It is instructive to imagine the three blocks of the same size and shape and painted the same colour; the application of an equal horizontal force to each, will at once reveal which is lead, which stone and which cork.

III. When the force applied to a certain mass is doubled or trebled, etc., then the acceleration produced is doubled or trebled etc.

IV. When the mass to which a certain force is applied is doubled, or trebled etc., then the acceleration is halved, or divided by three, etc.

V. Every force applied to a mass produces in that mass its proper acceleration in its own direction, independently of any other motion which the mass may have.

Thus, by III, Forces are measured by the number of celos which they respectively produce in 1 lb.

By IV, masses are measured by the inverse of the number of celos produced in them respectively by 1 poundal.

And by V, when two equal and opposite external forces act on a particle, they produce equal and opposite accelerations; in which case the particle continues at rest, or moving with constant velocity.

34. A force which is *applied* to a mass *A* by another mass *B* is said to be **external** to *A*.

35. Art. 31 gives a definition of *force* in terms of *mass*; in other words, Art. 31 states the **law**† which connects the ideas of *force* and *mass*.

It will be found to combine in one statement **Newton's first and second Laws of motion** [See Chapter XI].

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† Considered as a definition of *force* Art. 31 assumes that we know what *mass* is. We must in fact accept the definition of the *mass* of a body as the *quantity of matter* in it.

Suppose however we take our definition of **force** as the **push** *which when applied to a steel spring compresses it.*

We should in this case choose as our **unit** *force*, that *force* which applied to a certain spring compresses it in a certain manner.

The statement of Art. 31 would be arranged as a definition of mass as follows.

Mass is that which, when free from the application of any force, perseveres in its state of rest or of uniform velocity; and in which the application of force produces acceleration in the direction of the force, so that the mass of a body varies inversely as the acceleration produced in it by a given force, and varies as the force required to produce a given acceleration.

We should choose as our **unit** *mass* the mass in which the unit force produces unit acceleration.

36. The meaning of these definitions of Force and Mass may be stated thus: The motion of a given mass is always that of uniform velocity in a straight line, except during any interval in which it is acted on by external force. When external force acts on a mass its motion is that of acceleration in the direction of the force, so long as the force is in action, but no longer. The acceleration is proportional to the force which produces it; so that when the force changes the acceleration changes; when the force ceases the acceleration ceases, that is, the velocity becomes uniform.

By definition, 1 poundal acting on 1 lb. produces in it 1 celo.

Therefore 1 poundal acting *continuously* for 1 second on 1 lb. adds 1 velo to its velocity.

By definition 2 poundals acting on 1 lb. produce in it 2 celos.
and 2 poundals acting on 2 lbs. produce in it 1 celo.

Example i. *A force of* 3 poundals *is applied continuously to the mass* 1 lb. *which at the beginning of a certain interval has* 10 velos, *the force having the* same direction *as the velos: find* (i) *when the mass will have* 25 velos; (ii) how far *it will go in* 12 seconds.

By Art. 32, 3 poundals acting on 1 lb. produce in it 3 celos,
3 celos produce 15 velos in 5 seconds. (i)

The 1 lb. has 3 celos; the velocity of the 1 lb. at the middle of the 12 seconds is therefore $(10+3\times 6)$ velos; that is, 28 velos;
therefore in 12 secs. it passes over 28×12 ft., or, 336 ft. (ii)

Example ii. *A particle of* 4 lbs. *having* 256 velos *is acted on continuously by a* force *of* 32 poundals *in the direction* opposite to that of the velos; *find* (i) when *and* where *it will have* 224 velos, (ii) when and where *it will have* 224 velos in the opposite direction.

32 poundals acting on 4 lbs. produce in it $\frac{32}{4}$, that is 8, celos.

Hence the particle has -8 celos.

So that, if after x seconds the particle has 224 velos, we have
$224=256-8x$; whence, $x=4$.
In these 4 seconds the particle passes over
4 times $(256-2\times 8)$ ft. or 960 feet. (i)

If after x seconds the particle has -224 velos,

$$\left.\begin{array}{l}\quad\text{we have } -224=256-8x\text{; whence, } x=\underline{60}.\\ \text{In these 60 seconds the particle passes over}\\ \quad 60 \text{ times } (256-30\times 8) \text{ ft. or } \underline{960\text{ ft.}}\end{array}\right\}\text{(ii)}$$

Thus, the particle after 4 seconds has 224 velos; it continues in the same direction until the action of the force, producing in it a negative acceleration, brings it to rest; then the action of the force gradually produces in it velocity in the opposite direction, and it will be found that, as it returns, *it repasses each point with the same velocity in the opposite direction*. [See Lock's Elementary Trigonometry cap. VIII.]

EXAMPLES. X.

NOTE. *In these and in all future examples, in which a force is applied to a mass, it is supposed that the force is so applied, that the* shape *of the mass need not be considered; in some cases, the body is supposed to be so* small *that its* size *need not be considered; such a body is called a* **particle**.

Find the acceleration in each of the following 6 cases.

1. 3 poundals acting on 6 lbs.

2. 32 poundals acting on 1 lb.

3. 32 poundals acting on 64 lbs.

4. 64 poundals acting on 5 lbs.

5. 48 poundals acting on 1 cwt. [=112 lbs.]

6. 7 poundals acting on 4 cwt.

Find the velocity acquired and the distance passed over in each of the 6 following cases where the mass is *initially at rest.*

7. 3 poundals acting on 6 lbs. for 10 seconds.

8. 12 poundals acting on 1 lb. for 4 secs.

9. 18 poundals acting on 27 lbs. for 3 secs.

10. 32 poundals acting on 56 lbs. for 9 secs.

11. 56 poundals acting on 2 cwt. for 12 secs.

12. 64 poundals acting on 2 lbs. for $\frac{1}{2}$ sec.

13. Find the velocity acquired and the distance passed over in each of the 3 following cases, the force having the same direction as the initial velocity.

(i) 4 poundals acting for 3 secs. on 3 lbs. having initially 4 velos.
(ii) 5 poundals acting for 4 secs. on 10 lbs. having initially 20 velos.
(iii) 22 poundals acting for $3\frac{1}{2}$ secs. on $\frac{1}{2}$ lb. having initially 4 velos.

14. 20 poundals act on a mass of 5 lbs. (initially at rest) for 3 seconds and then cease; how far will the mass go in 10 seconds more?

15. 64 poundals act for 4 seconds on a mass of 2 lbs. initially at rest, after which the force is suddenly reversed; find how far the mass goes in 8 seconds from rest.

16. A particle of 1 lb. has 96 velos and is acted on by 32 poundals in the direction *opposite* to the velos; when will it have moved over 128 ft.? and find its velocities at those times.

17. A particle of 1 lb. has 64 velos, and is acted on by 32 poundals in the direction opposite to the velos; when will it be at rest? when will it be moving in the opposite direction with 32 velos?

18. A particle of 1 lb. acted on by a constant force moves in a certain second over 20 ft., and in the next second but one it moves over 128 ft.; find the force.

19. A particle of 1 lb. is at rest, when being acted on by a constant force, it moves over 16 ft. in the first second of its motion; find the force.

20. A mass of 20 lbs. is acted on by a constant force, and in the fifth and seventh seconds of its motion it passes over 108 ft. and 140 ft. respectively: find its initial velocity and the magnitude of the force.

21. A particle acted on by a constant force of 20 poundals passes over 72 ft. while its velocity increases from 16 to 20 velos; find its mass.

22. A particle acted on by 20 poundals has a velocity of 45 miles per hour after 12 seconds from rest; find its mass.

23. A train of 90 tons on a horizontal plane at rest is acted on by a force of 1000 poundals; the force acts for one minute and then ceases; find how far the train travels before coming again to rest, supposing the friction of the rails etc. to be equivalent to a retarding force of 300 poundals. [N.B. 1 ton = 2240 lbs.]

24. A train of 100 tons attains from rest a velocity of 40 miles per hour in going 1 mile; the breaks are then applied and the train comes to rest in 440 yds. Compare the retarding force exerted by the breaks with the force exerted by the engine, (neglecting the rotary motion, and the ordinary friction of the wheels etc.).

25. A force of p poundals acts for n minutes on m lbs.; find the velocity generated.

26. Find how many poundals will produce in 10 lbs. a velocity of 30 velos after going 25 yards from rest.

27. A train of 100 tons is impelled along a horizontal railroad by a horizontal force of 15040 poundals; neglecting friction etc., how long will it take to go 4 miles from rest? and how many miles per hour will it be going at the end of 4 miles?

37. The force which produces a celos in m lbs. is ma poundals.

For, to produce 1 celo in mass 1 lb. 1 poundal is required;
to produce 1 celo in mass m lbs. m poundals are required;
to produce a celos in mass m lbs. ma poundals are required.

Example. A force f poundals *acts continuously on a particle* m lbs. *which at a certain instant has* u velos *in the direction of the force; find* (i) *the* acceleration, (ii) *the* velocity *after* t seconds, (iii) *the* distance *it passes over in* t seconds, (iv) *how many* seconds *it takes to go* s feet.

1 poundal acting on 1 lb. produces 1 celo.
f poundals acting on 1 lb. produces f celos.
f poundals acting on m lbs. produce $\underline{\frac{f}{m} \text{ celos.}}$ (i)

In t seconds $\frac{f}{m}$ celos *produce* $\frac{ft}{m}$ velos, therefore after t seconds the velocity is $\underline{\left(u+\frac{ft}{m}\right) \text{ velos.}}$ (ii)

the velocity at the middle of the interval of t secs. is $\left(u+\frac{1}{2}\frac{ft}{m}\right)$ velos, and therefore the distance passed over in t seconds is

$$\underline{\left(ut+\tfrac{1}{2}\frac{ft^2}{m}\right) \text{ feet.}} \qquad \text{(iii)}$$

Let the particle take x seconds to pass over s feet, then by (iii)

$$s=ux+\tfrac{1}{2}\frac{fx^2}{m},$$

from which quadratic equation x may be found. (iv)

EXAMPLES. XI.

1. How many poundals are required to produce g celos in m lbs.?

2. How far will a ton [2240 lbs.] acted on by f poundals go from rest in t seconds?

3. How many poundals are required to produce g celos in k tons?

4. 1000 poundals acts upon a train of k tons on a horizontal plane; the friction of the rails etc. is equivalent to a retarding force of 250 poundals, how long will the train take to get up from rest a speed of m miles per hour?

38. DEF. The **mass-velocity** (or, the **momentum**) of a particle is that which varies as the mass and also as the velocity of the particle.

We shall choose as our **unit** *mass-velocity* that of a particle of 1 lb. moving with 1 velo.

We shall call this unit a **pound-velo.**

It follows that the *number* of *pound-velos* which a particle has, is the product of the *number* of lbs. in it by the *number* of velos with which it is moving.

39. DEF. The **mass-acceleration** of a particle is that which varies as the mass and also as the acceleration of the particle.

We shall choose as our **unit** *mass-acceleration* that of a particle of 1 lb. moving with 1 celo.

We shall call this unit a **pound-celo.**

It follows that the *number* of *pound-celos* which a particle has, is the product of the *number* of lbs. in it by the *number* of celos with which it is moving.

It should be noticed that the **measure** of the mass-acceleration of a particle is the **rate** of increase of its mass-velocity.

Example i. *Find the* mass-velocity *of a train of* 120 tons *going* 30 miles an hour.

$$120 \text{ tons} = 120 \times 20 \times 112 \text{ lbs.}$$

$$30 \text{ miles an hour} = 44 \text{ velos.}$$

Therefore, the mass-velocity of the train is

$120 \times 20 \times 112 \times 44$ pound-velos, that is 11,827,200 pound-velos.

Example ii. *Find the* mass-acceleration *of a* man and bicycle *of* 20 stone *while attaining a speed of* 15 miles an hour *in going* 121 ft. *from rest.* [1 stone = 14 lbs.]

15 miles an hour = 22 velos. [Ex. ii. p. 3.]

We have $2as = v^2$,

or, $2a \times 121 = (22)^2$; therefore, $a = 2$.

Also 20 stone = 20×14 lbs.

Therefore, the required mass-acceleration is 560 pound-celos.

EXAMPLES. XII.

Find the mass-velocity (or, momentum) of each of the following:

1. A mass of 3 lbs. having 3 velos.

2. A mass of 1 cwt. having 10 velos.

3. A train of 100 tons going at the rate of 30 miles an hour.

4. A bullet of 1 ounce going 1000 ft. per second.

5. A cannon ball of 1 cwt. going 1000 yds. per second.

6. A man of 12 stone running 12 miles an hour. [a stone = 14 lbs.]

Find the mass-acceleration of each of the following:

7. A mass of 10 lbs. having 8 celos.

8. A mass of 2 cwt. having 32 celos.

9. A train of 100 tons which has gained from rest a velocity of 30 miles per hour in 1 minute.

10. A bullet of 1 oz. which in traversing 36 inches from rest has gained a velocity of 1000 ft. per second. [16 oz. = 1 lb.]

11. A cannon ball of 56 lbs. which in traversing the tube of a cannon 10 ft. long, from rest, has gained a velocity of 500 yards per second.

12. A skater of 12 stone, who starting from rest has acquired a velocity of 20 miles an hour in going 50 yds.

40. It follows from Art. 31 that the number of poundals in an external force acting on a mass, is equal to the number of pound-celos which it produces in that mass.

Example i. *A force* 32 poundals *is applied to a ton;* find (i) *the* acceleration *produced,* (ii) how long *this force will take to produce* 6 velos *in the ton.*

Let the number of celos produced by the 32 poundals be a;
then, the mass acceleration produced is $2240a$ pound-celos.

By Art. 40 the external force which produces $2240a$ pound-celos is

$$2240a \text{ poundals.}$$

But, this force is $\quad 32$ poundals.

Therefore, $\quad 2240a$ poundals $= 32$ poundals;

that is, $\quad 2240a = 32$; or, $a = \frac{1}{70}$.

Thus, the acceleration produced is $\underline{\frac{1}{70} \text{ of a celo.}}$ (i)

$$\tfrac{1}{70} \text{ celo in 1 second produces } \tfrac{1}{70} \text{ velo,}$$

$$\tfrac{1}{70} \text{ celo in } 6 \times 70 \text{ seconds produces 6 velos.}$$

Therefore, if the force be continuously applied to the ton it will produce 6 velos $\underline{\text{in 6 minutes.}}$ (ii)

Example ii. *A cannon ball of* 250 lbs. *starting from rest, leaves the mouth of a cannon with a velocity of* 1000 ft. per second; *find the resultant force which acted upon it when in the cannon, supposing this force to be uniform and that the ball passed over* 20 *ft. while in the cannon.*

[NOTE. By *resultant* force is meant, the force over and above that necessary to overcome the friction of the tube; the pressure of the gases on the base of the shot is necessarily greater than this resultant force, which is only that part of this pressure which produces the acceleration of the shot.]

We have to find the number of celos which the ball had in the cannon.

Here, $v=1000$, $s=20$ ft., $u=0$,

and $2as=v^2$.

Therefore, $40a=1000000$; or, $a=25000$.

The mass-acceleration is ma;

that is, 25000×250, or 6250000 pound-celos.

The number of poundals required to produce 6250000 pound celos is equal to the number of pound-celos produced,

that is =6250000 poundals.

EXAMPLES. XIII.

1. Find the number of poundals required to produce the motion in each of the examples **7** to **12** of Examples XII.

2. Find the number of poundals which by acting for 20 seconds will produce the mass-velocities in Questions **1** to **6** of Examples XII, respectively.

3. A locomotive engine can exert upon a train of 100 tons a force of 10000 poundals more than is necessary to counteract the friction; how long would this engine take in working the train on a level line up to a speed of 30 miles an hour?

4. A rifle bullet of 1 oz. leaves the barrel of a rifle which is 2 ft. 6 in. long with a velocity of 1200 feet per second; find the resultant force, supposed uniform, with which it was acted on in the barrel.

5. A mass of 10 lbs. moves over 16 ft. from rest in 2 seconds; supposing its motion to be that of uniform acceleration, what force is acting on it?

6. It is observed that 1 ton is moving in a straight line with a velocity of 10 miles per hour; 3 minutes later it is observed to be moving in the same straight line with a velocity of 12 miles per hour; supposing a uniform external force to have acted on the mass during those 3 minutes, how many poundals did it contain?

WEIGHT.

41. DEF. The **weight** of a mass is the *force* with which it is attracted by the earth.

When we support a mass in our hand so as to keep it at rest, we can only do so by the application of an upward push or force; this upward *force* is equal and opposite to the *weight* of the mass.

42. Since the weight of a mass is a force, and since our unit of force is a poundal, in order to measure the weight of a given mass we must express its weight in poundals.

The number of poundals in any force is equal to the number of pound-celos which it produces; accordingly we have to ascertain how many celos the weight of any particular mass produces in that mass.

43. The following experiment is important.

A chamber, such as the receiver of an air-pump, is exhausted of air. In the chamber are placed several masses of different kinds and of different sizes; for instance, a leaden bullet, an iron nail, a bronze penny, a gold sovereign, a glass ball, a piece of cork, a feather; these are all placed on a ledge so arranged that they can be simultaneously let fall.

When a mass is 'let fall' in a vacuum, the *only* force acting upon the mass is its own weight; accordingly in this experiment when the masses are let fall, the weight of each mass produces in its own mass motion, in a vertical direction downwards, which is that of uniformly increasing velocity.

Now it is observed that, having been simultaneously let fall from the same height, these different masses all reach the bottom of the chamber *simultaneously*.

But, when a number of points, starting from rest and each moving in the same direction with uniformly increasing velocity, pass over equal distances in the same interval of time, they must all be moving with the *same acceleration*. Hence,

44. The *weight* of a mass is a *force*, which produces in its own mass a certain definite acceleration; which accelera-

tion is the same for all masses of whatever magnitude and of whatever kind of material.

45. This definite acceleration, which is called the **acceleration due to gravity**, is different at different places on the earth's surface.

It is greatest near the Poles, (where it is about 32·255...celos) and it gradually diminishes to about 32·091...celos at the equator.

It is the same at places having the same latitude.

It has been ascertained by observation that at Greenwich the number of celos produced in any mass when falling *freely* towards the earth's surface (that is, when moving under the action only of its own weight) is 32·19....

46. It is usual to use the letter g to indicate the *number* of units of acceleration produced in a mass by the action of its own weight.

Hence, at Greenwich $g = 32{\cdot}19\ldots$

The acceleration produced in a mass m lbs. by its own weight is g celos; hence the amount of mass-acceleration produced in mass m lbs. by the weight of m pounds is mg pound-celos; the *force* required to produce mg pound-celos is mg poundals. Therefore

the weight of m lbs. is the *force* **mg poundals.**

N.B. In all numerical examples we shall use
32 poundals as a sufficient approximation to
the **weight** of 1 lb.

Example i. *How many* poundals *are there in* the weight of 3 lbs.?

The weight of 3 lbs. produces in the mass 3 lbs. (about) 32 celos.

Therefore the *weight* of 3 lbs. is equivalent to the *force* 3 × 32 poundals.

Example ii. *How many* celos *will a force equal to* the weight of 3 lbs. *when applied to a* mass 12 lbs. *produce in it?*

The force is 3 × 32, or, 96 poundals
and 96 poundals produces in 12 lbs., $\frac{96}{12}$ or 8 celos.

Example iii. *How long does a stone take in falling from rest under the action of gravity through a vertical distance of* 128 ft.?

By Arts. 43—46 the stone has an acceleration of 32 celos downwards.

Let t seconds be the time of passing over 128 ft.;

then, $128 = \frac{1}{2} \times 32t^2$;

or, $t^2 = 8$;

therefore, $t = 2\sqrt{2} = 2 \times 1{\cdot}4142 = 2{\cdot}8284\ldots$

The interval occupied is therefore 2·8... seconds.

Example iv. 16 lbs. *is acted on continuously by a force equal to the weight of* 4 lbs.; *find its* acceleration; *and find how far it moves from rest in* 10 seconds.

The weight of 4 lbs. $= 4 \times 32$ poundals.

This force 4×32 poundals acting on 16 lbs. produces in it $\frac{4 \times 32}{16}$ celos;

That is, this force produces in 16 lbs. the acceleration 8 celos.

In 10 seconds the 16 lbs. will pass over a distance s feet such that

$$s = \tfrac{1}{2}at^2 = \tfrac{1}{2} \times 8 \times 100 = 400.$$

The distance required is 400 feet.

NOTE. A force is said to be expressed *in pounds weight*, when we state the number of lbs. whose *weight* is equal to the force.

Example v. A mass 4 cwt. *is observed to be moving with* 16 celos; *express* in pounds weight *the* force *which is acting upon it.*

When 4 cwt. has 16 celos, it has $4 \times 112 \times 16$ pound-celos.

To produce this, $4 \times 112 \times 16$ poundals must be acting upon it; $4 \times 112 \times 16$ poundals are equal to the weight of $\frac{4 \times 112 \times 16}{32}$ lbs.; that is, the weight of 2 cwt. must be acting on the 4 cwt.

EXAMPLES. XIV.

1. A stone is let fall from the top of a tower 256 ft. high; in how many seconds will it reach the ground?

2. A stone is let fall; how far will it fall (i) in the 2nd second, (ii) in the 4th second from rest?

3. A stone is thrown vertically upwards with a velocity of 320 velos; when will it come to rest? and how high will it go?

4. A stone is thrown vertically upwards and returns after 4 seconds to the point from which it was thrown; with what velocity was it thrown? and how high did it go?

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† Example iv. p. 16 and Example i. p. 20 should be noticed.

5. A stone is thrown vertically upwards and passes a point 48 ft. high with a velocity of 32 ft. per second; in how many seconds will it return to the point of projection?

6. A stone is let fall from a tower 256 ft. high and at the same instant another stone is projected from the foot of the tower with just sufficient velocity to carry it to the top of the tower; when and where will the stones meet?

7. A body of 1 cwt. is acted on by a force equal to the weight of 7 lbs.; find its acceleration.

8. A train of 100 tons is drawn with an acceleration of 4 celos; express in pounds weight the force exerted by the engine in addition to that necessary to overcome the friction.

9. 28 lbs. placed on perfectly smooth ice is acted on by a horizontal force equivalent to the weight of 1 lb.; find its velocity after 7 minutes.

10. Two masses consisting of a cubic foot of lead and a cubic foot of iron placed at rest on perfectly smooth ice are each acted on in the same direction by a horizontal force equal to the weight of 1 lb.; how far will they be apart after 30 seconds?

[NOTE. A cubic ft. of lead weighs about 11340 oz.; a cubic ft. of iron weighs about 7680 oz.]

11. 1 ton on a smooth horizontal plane is observed to move from rest a distance of 48 ft. in 60 seconds; if the motion is caused by a uniform horizontal force, how many lbs. would the force lift?

12. A train of 100 tons is drawn by an engine which can exert a force, which would lift a weight of 800 lbs.; supposing the force of the friction etc. to be equivalent to a force of 100 lbs. weight, find how long the train would be in attaining from rest a velocity of 60 miles per hour.

13. A mass of 1 cwt. is placed on ice and is acted on by a horizontal force equal to the weight of 8 lbs.; supposing that a force equal to the weight of 1 lb. is sufficient to overcome the friction of the ice, find when the mass will have 100 velos.

14. Supposing the force of 8 lbs. weight of Question **13** to be withdrawn when the mass is moving at the rate of 60 miles an hour, in how many seconds will the mass be brought to rest by the retardation due to the friction?

47. In Art. 28 it is assumed that we can apply a constant force to a mass.

Such a force might be obtained by taking care that the force is just sufficient to keep a certain steel spring compressed in a certain manner.

By applying the same constant force to each of two bodies in turn and observing the number of celos produced in them, we obtain a measure of the mass of the bodies.

It happens however that the experiment of Art. 43 provides a more convenient method of measuring the mass of a body. For, since the weight of a mass is a force which produces a fixed number (g) of celos in the mass itself, the *weight* of a mass is proportional to the mass itself; therefore, **equal masses have equal weights.**

By a *ton* of coal is meant a certain amount of mass.

That a certain quantity of coal contains a ton mass is ascertained by weighing it; that is, we ascertain that the mass of coal has the same weight as a certain other standard mass of iron, which mass we call a ton.

NOTE. In the language in common use, the ideas of *mass* and *weight* are not clearly distinguished.

For example, a piece of metal is said to be

'*a* 7 lbs. *weight*';

here, the word *weight* means, *a piece of metal whose mass is* 7 lbs., *whose* weight *is used for the purpose of comparing the* weights *of other masses.*

Or again, 'a train *weighing* 100 tons';

here, the word *weighing* means, '*whose* weight *is equal to that of* 100 tons.'

Again, a string is often said to be *light*, meaning *weightless*, or *of too small* **weight** *to be observed;* such a string must also *be of too small* **mass** *to be observed.*

The student however must carefully remember that

the **unit mass** is 1 lb.,

while **the weight of 1 lb. is 32·2....poundals.**

48. The two following examples should be noticed and the results compared.

Example i. *A mass of* 1 ton *lies on a smooth horizontal plane; a man by means of a windlass and string applies continuously to the mass a horizontal force equal to the weight of* 28 lbs.; *how long will he take to move the mass* 5 ft. *from rest.*

Let the force produce a celos in the ton; that is, in 2240 lbs.

The mass acceleration is therefore $2240a$ pound-celos.

To produce $2240a$ pound-celos the force required is

$$2240a \text{ poundals.}$$

The force actually applied is the weight of 28 lbs.;

which is, 28×32 poundals.

Hence 28×32 poundals $= 2240a$ poundals;

whence, $a = \frac{2}{5}$.

Let s ft. be the distance moved from rest in time t seconds; then,

$$s = \tfrac{1}{2}at^2;$$

here, $5 = \frac{1}{2} \times \frac{2}{5} \times t^2$;

or, $t^2 = 25$; therefore, $t = 5$.

The time required is 5 seconds.

Example ii. *A mass of* 1 ton *lies on a smooth horizontal table, and is attached by a light string which passes over the edge of the table, to a mass of* 28 lbs., *which hangs freely under the action of its own* weight; *how long will the masses take to move* 5 ft. *from rest?*

[NOTE: Here, the force acting on the ton mass is **not** equal to the weight of 28 lbs.; for

The force acting on the ton is the pull or *tension* of the string; and the tension of the string is *not* equal to the weight of the 28 lbs.; for if it were, the 28 lbs. mass would have acting on it (i) a pull equal to the weight of 28 lbs. *upwards* and (ii) its own weight (which is that of 28 lbs.) *downwards;* that is, it would have no *resultant* external force acting on it, and would therefore have *no acceleration.*

But, if the ton has acceleration, the 28 lbs. must have acceleration also.

Hence, the force acting on the ton, i.e. the tension of the string, is *less* than the weight of 28 lbs.; as will presently appear.]

I. Consider the external forces acting on the 28 lbs.

Let T poundals be the tension of the string; then the resultant external forces acting on the mass of 28 lbs. are its weight directly downwards and the pull of the string directly upwards,

that is, 28×32 poundals $-$ T poundals;

Let this resultant force produce a celos in the 28 lbs.

It produces therefore $28a$ pound-celos;

the force necessary to do this is $28a$ poundals; hence we see that

$$(28 \times 32 - T) \text{ poundals} = 28a \text{ poundals} \qquad \text{(i)}.$$

II. Consider the external force acting on the ton.

The weight of the ton does not concern the question; it is supposed to be entirely obliterated by the upward support of the smooth horizontal plane.

The only other force acting on the ton is the tension of the string. The string is supposed to be *light*, and therefore of no mass; so that i exerts the same pull on the ton as it does on the 28 lbs.; which pull is

T poundals.

Also, since the two masses are connected by an *inextensible* string, the acceleration of the ton is the same as that of the 28 lbs. viz., a celos.

The force necessary to produce a celos in 2240 lbs. is

$2240\,a$ poundals;

but, the force which produces the a celos is T poundals; hence we see that

$$T \text{ poundals} = 20 \times 112a \text{ poundals}; \qquad \text{(ii)}$$

whence, adding (ii) to (i) $28 \times 32 = 81 \times 28a$;

therefore, $a = \frac{32}{81}$.

Let t seconds be the time in which the masses pass over 5 ft. from rest;

then, $5 = \frac{1}{2} \times \frac{32}{81} t^2$;

or, $t = \frac{9}{4}\sqrt{5} = \frac{9}{4} \times 2{\cdot}236\ldots$

$= 5{\cdot}031\ldots$

The time required is 5·031... seconds.

NOTE. Comparing these two *Examples* we observe that in *Example* i, the acceleration of the ton is $\frac{32}{80}$ celos, in *Example* ii the acceleration is *less*, viz. $\frac{32}{81}$ celos. In *Example* i the weight of 28 lbs. acts on a ton, in *Example* ii, the weight of 28 lbs. acts on (a ton + 28 lbs.).

EXAMPLES. XV.

1. A sledge weighing 1 ton is pulled on ice by a rope whose tension is equal to the weight of 56 lbs.; if the friction of the ice causes a horizontal retarding force on the sledge equal to $\frac{1}{50}$ of its weight find the acceleration produced.

2. A sledge, weighing 1 ton is pulled on ice by 4 dogs whose weight is 28 lbs. each; each dog obtains from the friction of the ice a horizontal force forwards equivalent to half its own weight; prove that if the friction of the sledge on the ice is $\frac{1}{50}$ of its weight the acceleration produced is $\frac{16}{105}$ celos.

3. A man of 12 stone on a railway truck weighing $1\frac{1}{2}$ tons propels it, when he is himself on it, by means of a rope fastened to a fixed point; he pulls with a constant force of 40 lbs. weight; compare the acceleration of the truck with that which he would produce if, fastening the rope to the truck and getting off, he pulled at the rope with a force of 40 lbs. weight; supposing that in each case a force of 12 lbs. weight is required to overcome the friction etc.

4. A mass of $1\frac{1}{2}$ tons placed on a smooth horizontal table is acted on by a horizontal force of 40 lbs. weight; find how far it would move from rest in 2 seconds.

5. A mass of $1\frac{1}{2}$ tons lies on a smooth horizontal table and is attached by a light string, which passes over the edge of the table, to a mass of 28 lbs. which hangs freely under the action of its own weight; how far will the masses move from rest in 2 seconds?

6. A man of 12 stone finds that he can in running attain a speed of 15 miles an hour in 3 seconds; find the horizontal force which he applies to his own mass, supposing that he attains that speed with uniform acceleration.

7. The man in Question **6** ties a string to a mass of 5 cwt. on a smooth horizontal sheet of ice and starts to run on the bank, also horizontal, using exactly the same horizontal force as in **6**; how long will he take to attain a speed of 15 miles an hour?

8. A train of 240 tons running at the rate of 30 miles an hour on a straight horizontal line comes to rest, when under the action of friction only, after running 4 miles; assuming the force of friction to be constant, find the force in lbs. weight which would just cause the train to move when at rest.

9. Express in lbs. weight the force which the engine of the train of Question **8** must exert that it may attain a velocity of 60 miles an hour in 1 min. 28 secs. from rest.

CHAPTER III.

INERTIA. STRESS.

49. DEF. The **inertia** or **inertness** of *mass* is that essential property by virtue of which mass persistently retains its existing velocity unaltered, so long as it is not acted on by external force.

The nature of *force* is such that it needs *time* in which to change the velocity of mass.

Mass is *inert*, in the sense that it cannot of itself make any *change* in its motion.

External force acting on mass produces in it acceleration only; it does *not* produce any definite *velocity* unless it acts *continuously* for some definite interval of time.

Example. When it is observed that a given mass M has moved in a given interval from a point A to another B, we can from this infer nothing as to whether an external force has acted on the mass during that interval or not. The mass M may have moved over that distance by reason of its inertia; by virtue of which it persisted in moving from A to B with uniform velocity.

But when it is observed that a mass M has at one instant a certain *velocity* and that after some interval it has a different velocity, we are able to state that, for some part at least of that interval, it has been acted on by some external force.

50. When two **equal forces** act simultaneously throughout some interval each on one of two masses M and M', they produce in each mass the *same number of pound-velos.*

The forces are not necessarily uniform, provided they are always equal; and when the masses M and M' are different the velocities produced will be different.

51. It happens that the action of two exactly equal forces, acting simultaneously for the same interval of time on different masses is a phenomenon which is constantly taking place.

52. No external force can be applied to a mass without the mass exerting an equal and opposite force on the body which applies the force. This is **Newton's Third Law.** '*To every action there corresponds an equal and opposite reaction.*' [See Chapter XI.]

53. DEF. **A stress** is that which consists of two equal and opposite forces together forming an action and its reaction.

Examples. The *pressure* of a heavy mass on the ground on which it rests, and the pressure of the ground on the mass, together form a stress.

The *tension* of a string at any point of its length is a stress.

54. When two particles act and react the one on the other, the stress causes no alteration in the *total* mass-velocity of the two particles; that is, the stress has no effect on the algebraical *sum* of their mass-velocities.

For a stress consists of two equal and opposite forces acting simultaneously; so that whatever amount of mass-velocity the action produces in one mass in any interval, the reaction produces in the other mass in the same interval an exactly equal amount in the opposite direction.

Example i. Let two particles A and B of m lbs. and m' lbs. be connected by a light string. The tension of this string is a stress; so that whatever force is applied by the string and for whatever interval to the particle A, an exactly opposite and equal force is applied for the same interval by the string to the other mass B. So that, if mv pound-velos is the *additional* mass-velocity produced in the course of any interval by the tension in A, and $m'v'$ pound-velos the *additional* mass-velocity produced in the same interval by the tension in B, then mv and $m'v'$ are equal and opposite; so that, $m'v' = -mv$;

or, $$mv + m'v' = 0.$$

Suppose for example that the masses always have the same velocity,

so that $$v = v',$$

then $$(m + m')\,v = 0;\ \text{therefore } v = 0.$$

Hence it appears that an internal stress between two particles, which cannot move relatively to each other, has no effect on their velocity.

Example ii. *Two masses* m lbs. *and* m′ lbs. *placed on a smooth horizontal plane are connected by a light inextensible string; a force of* f poundals *is applied to the mass* m *in the direction of the string, which is tight; find the* acceleration *produced; also the* tension *of the string.*

I. The masses are prevented by the string from moving relatively to each other; hence, we may consider the tension of the string as an internal stress, and consequently the two masses as a single mass.

Thus, we have a mass of $(m+m')$ lbs. acted on by f poundals; therefore the acceleration a is given by the equation $f=(m+m')\,a$;

that is, the acceleration is $\frac{f}{m+m'}$, celos (i).

II. To find the *tension* of the string we must consider each component of the stress separately.

Let the string apply an external force $-T$ poundals to the mass m; it will consequently apply an external force T poundals to the mass m'.

Then, if a celos be the acceleration (which is the same for each mass, for the string is tight and inextensible)

we have, $f-T=ma$; [from the motion of m

and, $T=m'a$; [from the motion of m'

whence, by addition, $f=(m+m')\,a$ (i),

wherefore $T=\frac{m'}{m+m'}f$.

The tension required is $\frac{m'}{m+m'}f$ poundals (ii).

EXAMPLES. XVI.

1. An engine of 40 tons is attached to a train of 6 carriages of 10 tons each; the train is moving with 5 celos; neglecting friction and the rotatory motion of the wheels, find the tension of the coupling between the engine and the next carriage.

2. In the train in Question **1** find the tension of the coupling between each pair of carriages.

3. In the train in Question **1**, what force is the engine exerting?

4. The train in Question **1** moving at the rate of 30 miles an hour is stopped by brakes on the engine in a quarter of a mile; find

(i) the force exerted by the brake,

(ii) the stress between the engine and the next carriage,

(iii) the stress between the two last carriages,

(iv) if the brakes on the engine exerted the same force in all cases, how far would the engine alone, going 30 miles an hour when the brakes are applied, run before coming to rest?

5. An engine which exerts a horizontal force sufficient to generate an acceleration a celos in its own mass m lbs., is attached to 3 carriages whose masses are $\frac{1}{2}m$ lbs., $\frac{1}{4}m$ lbs. and $\frac{1}{8}m$ lbs. respectively; find the acceleration of the train and the stress between each carriage, neglecting friction and the rotary motion of the wheels.

6. An engine which exerts a horizontal force p poundals, sufficient to generate in its own mass a celos, is attached to n carriages each of which is $\frac{1}{10}$th of the mass of the engine; find the acceleration of the train and the stress between the 4th and 5th carriages, neglecting friction etc.

55. The laws of motion may be illustrated and tested experimentally by the aid of **Atwood's machine.**

56. Atwood's machine may be described as follows.

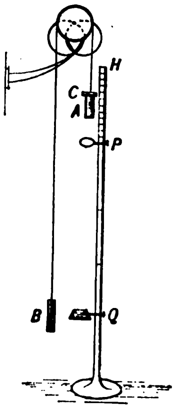
A wheel or pulley, made as light and as free from friction as possible, is fixed at a convenient height from the ground; over the pulley is passed a light inextensible string, usually made of silk; from the ends of the string are suspended two masses M and M'. Each mass is acted on only by its own weight and by the tension of the string.

I. It will be found that we can arrange the masses so that they move with a very *small* acceleration compared with that produced in a mass falling freely under the action of its own weight; so that we can ascertain by actual measurement what *kind* of motion is produced by force in mass; we can thus test the statement that force produces in mass that kind of motion which has been defined as uniformly increasing velocity.

II. We can test by observation the statement that mass when acted on by no resultant external force moves with uniform velocity.

III. Since the mass of the string is very small, it causes no appreciable effect on the tension; if we also neglect the effects produced by the mass of the wheel and the friction of the axle (which are both comparatively small), then the tension remains unchanged throughout the whole length of the string. The string may then be described as *the means of communicating* the two portions of a stress which act one on each of the masses; in this way, by comparing the actual motion of the masses with the motions given by our calculations, we have a practical test of the truth of the Third Law of Motion.

Example i. *In an* Atwood's machine *the masses suspended* A *and* B, *each contain* M lbs.; *a small mass* C *of* m lbs. *is placed on the mass* A; *find the resulting* acceleration *and the* tension *of the string.*



I. Consider the motion of the masses (A and C), treating them as one mass.

The external forces acting are

(i) the weight of $(M+m)$ lbs.;
that is, $(M+m)g$ poundals,
(ii) the tension of the string; let this be
T poundals.

The weight acts vertically downwards and the tension acts vertically upwards; hence, the resultant external force is

$$\{(M+m)g-T\} \text{ poundals } \textit{downwards},$$

Let (A and C) have α celos downwards;
then it has $(M+m)\alpha$ pound-celos;
so that the force acting upon it must be
$(M+m)\alpha$ poundals downwards.

Therefore, $(M+m)g-T=(M+m)\alpha$ (i).

II. Consider the motion of the mass B.

The string being inextensible, whatever motion A may have, B must have an equal motion in the opposite direction;
therefore B has an acceleration $-\alpha$ celos downwards.

The mass acceleration of B is therefore

$$-M\alpha \text{ pound-celos, } \textit{downwards}.$$

So that it is acted on by $-M\alpha$ poundals.

The resultant external force is $(Mg-T)$ poundals, *downwards*.

For the tension of the string is a stress which acts equally on A and B in opposite directions.

Therefore $$Mg - T = -Ma \qquad \text{(ii).}$$

Hence, by subtraction of (ii) from (i),

$$mg = (2M + m)\,a,$$

that is, $$a = \frac{m}{2M + m}g \qquad \text{(iii).}$$

Also from (ii) $T = Mg + Ma, = \dfrac{2M(M+m)}{2M+m}g.$

Example ii. *In an* Atwood's machine *the masses* A *and* B *are each* $15\frac{1}{2}$ oz. *and a mass* C *of* 1 oz. *is placed gently on the mass* A *when at rest; after* 2 seconds *C is removed; discuss the resulting motion.*

I. Consider the motion of the masses A and C, considering them as one mass.

The external forces are its weight, which is $\frac{33}{32} \times 32$ poundals, and the tension, T poundals; let a celos downwards be its acceleration; then its mass-acceleration is

$$\tfrac{33}{32} \times a \text{ pound-celos};$$

whence, as in Ex. i, $$33 - T = \tfrac{33}{32}a \qquad \text{(i).}$$

II. Consider the motion of the mass B; then as before and as in Example i, we have

$$31 - T = -\tfrac{31}{32}a \qquad \text{(ii).}$$

Whence by subtraction

$$2 = \tfrac{64}{32}a = 2a,$$

or $$a = 1 \qquad \text{(iii).}$$

Thus, as long as the mass C forms part of the mass A each mass has 1 celo, the mass B upwards and the masses A and C downwards.

Hence at the end of 2 seconds each mass will have 2 velos and will have moved over ($\frac{1}{2}at^2$), that is, 2 feet.

Forming the equations (i) and (ii) on the supposition that C is not put upon A, we have,

$$\left.\begin{aligned} 31 - T &= \tfrac{31}{32}a \qquad &\text{(i)} \\ \text{and,}\quad 31 - T &= -\tfrac{31}{32}a \qquad &\text{(ii)} \end{aligned}\right\}$$

whence $$a = 0.$$

So that when the 1 oz. mass is removed the acceleration vanishes; that is, the velocity is uniform; and the masses each move with 2 velos, A downwards and B upwards.

Example iii. *Find the* stress *between the small* mass C *and the* mass A *in Example* ii *of page* 47.

The mass A and the mass C each descends with 1 celo.

The weight of the mass C, viz. 2 poundals acting on $\frac{1}{16}$ lb. would produce in it 32 celos.

It actually has 1 celo.

Hence the stress is such that the upward part of it acting on $\frac{1}{16}$ lb. destroys in it 31 celos.

The upward action of the stress is therefore $\frac{31}{16}$ poundals.

Example iv. *A horizontal platform is made to descend vertically with* 4 celos; *a man stands on the platform with* 56 lbs. *in his hand; shew that this mass appears to weigh only* 49 pounds.

The force acting on the 56 lbs. (besides the pressure of the hand) is its weight downwards, that is,

$$56 \times 32 \text{ poundals,}$$

This would produce in it 32 celos, if the upward force of the man's hand did not prevent. The 56 lbs. actually has 4 celos, which requires

$$56 \times 4 \text{ poundals to produce it.}$$

Thus, of the weight (56×32 poundals) the part 56×4 poundals are taken up in producing the 4 celos; so that the upward push of the man's hand must counteract the remainder. In other words this push is

$$56 \times 28 \text{ poundals, which is the weight of 49 lbs.}$$

57. To find the acceleration in an Atwood's machine we may proceed as follows.

Let the masses be m lbs. and m' lbs.

Then, considering the string as the means of preventing relative motion between the masses, and therefore the mass moved to be the sum of the masses m lbs. and m' lbs., the tension of the thread becomes an internal stress and has no effect on the mass-acceleration produced.

The resultant external force in the direction of the motion of the particles is (the weight of the greater mass m − that of the less m'), hence we have $mg - m'g = (m + m')\,a.$

58. The problem of Atwood's machine is similar to the following.

Two particles A and B of masses m lbs. and m' lbs., connected by a light inextensible string, are placed with the string stretched, on a smooth horizontal plane; A is acted on by a horizontal force mg poundals in the direction BA, and B is acted on by a horizontal force $m'g$ poundals in the direction AB; find the tension of the string and the acceleration of the masses.

It will be found that the tension is $\frac{2mm'}{m+m'}g$ poundals; and that the acceleration is $\frac{m-m'}{m+m'}g$ celos.

EXAMPLES. XVII.

Find the tension of the string and the acceleration of the masses in an Atwood's machine, when the masses A and B have the following values:

1. Mass of A, 1 lb. 1 oz.; of B, 15 oz.

2. Mass of A, 5 lbs.; of B, 3 lbs.

3. Mass of A, 25 oz.; of B, 23 oz.

4. Mass of A, 4 lbs. 4 oz.; of B, 3 lbs. 12 oz.

5. In an Atwood's machine the masses A and B are 3 lbs. 15 oz. each; a mass of 2 oz. is gently placed on A and is removed by a catch after A has descended 1 ft.; shew that A will take 4 seconds to descend the next 4 ft.

6. In an Atwood's machine the masses are each 7 lbs. 15 oz.; a mass of 2 oz. is gently placed on one of them and is removed by a catch after descending 8 ft.; how long will it take to pass over the next foot?

7. In an Atwood's machine it is observed that after descending 8 ft. from rest the masses have a velocity of 4 ft. per second; the larger mass is 32½ oz.; find the other mass.

8. In an Atwood's machine the masses are equal and it is observed that a weight of 1 oz. added to one of them produces 1 celo; what are the masses?

9. In an Atwood's machine the pulley is suspended by a hook from a nail; shew that (with the usual notation) the force on the nail is $\left(\frac{4mm'g}{m+m'}\text{ poundals} + \text{the weight of the pulley}\right)$.

10. Shew that in an Atwood's machine the tension of the string diminishes as the ratio of the larger mass to the smaller increases, their sum remaining unaltered.

11. Prove that in an Atwood's machine when the tension of the string is one fourth of the sum of the weights $m : m' = 3 + 2\sqrt{2} : 1$.

12. Two masses m lbs. and m' lbs. connected by an inextensible thread, are placed with the thread straight on a smooth horizontal plane; the mass m is acted on in the direction of the string away from m' by a force f poundals, and m' is acted on by a force f' poundals in the opposite direction; find the acceleration of the masses.

13. Prove that if in Ex. **12** the forces are equal to the weights of the masses on which they act, the acceleration is $\frac{m-m'}{m+m'}g$ celos.

14. A bullet fired directly into a block of wood will penetrate 3 in.; what part of its velocity will it lose in passing through a board of the same wood 1 in. thick supposing the resistance uniform?

15. A mass of 1 cwt. is placed on a lift which descends vertically with a uniform acceleration of 10 celos; find the stress between the lift and the mass.

16. If the lift in Question **16** is ascending vertically with an acceleration 16 celos, what is then the pressure?

17. A man can raise a sack of corn weighing $1\frac{1}{2}$ cwt.; shew that when on a lift which is made to descend vertically with 8 celos, he can raise 2 cwt.

18. A balloon ascends vertically with a uniformly accelerated motion so that a weight of 1 lb. produces, on the hand of the aeronaut sustaining it, a downward pressure equal to that which 17 oz. produces when at rest; find the height which the balloon attains in 1 min. from rest.

19. Find the pressure on the mass of 2 oz. in Question **5.**

20. Find the magnitude of the stress between the 2 oz. and the mass on which it is placed in Question **6.**

CHAPTER IV.

IMPULSE. IMPACT. ELASTICITY.

59. We have defined *force* as that which when applied to mass produces *acceleration*.

No force however great can produce in a mass a finite addition of velocity unless the force acts (producing acceleration in the mass) throughout a finite interval of time.

60. DEF. An **impulse** is that which acting on a mass produces in it a finite velocity, so that the mass-velocity produced varies as the impulse.

61. DEF. We shall choose as our **unit** *impulse* that impulse which acting on a pound produces in it 1 velo.

We shall call the *unit impulse* a **pulse**.

It should be noticed that in the definition of an impulse nothing is said about the length of the interval occupied by the impulse.

An impulse is quite a different thing from a force—just as different as a velo is from a celo.

Examples. Each entire stroke of a tricyclist's foot on the pedals is an impulse.

Each complete down stroke of the handle of a pump is an impulse.

62. An impulse is the equivalent of a force acting for some interval.

A pulse has the same effect on a mass as a poundal acting throughout 1 second; each produces a pound-velo in the mass on which it has acted.

It has the same effect as (i) 2 poundals acting for $\frac{1}{2}$ second; (ii) 100 poundals acting for $\frac{1}{100}$th of a second; (iii) 10000 poundals acting for $\frac{1}{10000}$th of a second; and so on.

63. When a mass has received a finite addition to its velocity we may say either (i) that it has received an impulse; or (ii) that a force has acted on it for a finite interval.

Example. A shot of 8 cwt. *from a* 38 ton gun *is observed to leave the gun with a velocity of* 2000 velos; (i) *find the* impulse *it has received;* (ii) *if the shot passes over* 10 ft. *when in the tube of the gun, find the resultant force which has acted on the base of the shot, supposing that force to have been* uniform.

The shot has received $2000 \times 8 \times 112$ pound-velos.

Therefore $2000 \times 8 \times 112$ pulses have acted on it. (i)

The force which produced this mass-velocity was uniform; so also therefore must have been the acceleration; also the shot passed over 10 ft. while in the gun.

Let the acceleration be a celos,

then, [Art. 26, iii] $a \times 10 = \frac{1}{2}v^2 = \frac{1}{2} \times (2000)^2$,

whence, $a = 200000$.

The mass of the shot is 8×112 lbs.; so that its mass acceleration is

$8 \times 112 \times 200000$ pound celos.

To produce this, $8 \times 112 \times 200000$ poundals are necessary.

The force is therefore 179200000 poundals,

equivalent to the weight of $\frac{179200000}{32 \times 20 \times 112}$ tons; that is, of $312\frac{1}{2}$ tons. (ii)

*64. The effect of a large force acting during a short interval and that of a small force acting for a large interval may each be equal to the effect of the same impulse.

Thus an impulse requires two things to produce it (i) an interval of time, (ii) a force.

A *pulse* might be termed a *poundal-second;* meaning that a pulse has the same effect as a poundal acting for a second.

Suppose the interval occupied by q pulses is t seconds; q pulses produce q pound velos. Also, $\frac{q}{t}$ poundals acting throughout t seconds produce q pound velos; therefore, the average force which produces q pulses in t seconds is

$$\frac{q}{t} \text{ poundals.}$$

Hence, the measure of the average force which produces a given impulse is the ratio of the number of pulses acting to the number of seconds required.

In other words *the average force producing an impulse is measured by the* **rate** *of impulse per second.*

Example. A glass ball of 4 oz. *strikes a pavement with* 50 velos, *and is observed to rebound with* 30 velos; *find the* impulse *which it receives; supposing the* impulse *occupies* $\frac{1}{100}$th of a second, *find the* average magnitude *of the* stress *between the ball and the pavement.*

[NOTE. When a glass ball is let fall upon a horizontal pavement the velocity on reaching the pavement seems to be very quickly changed.

The motion of the pavement itself is imperceptible (for a reason explained in Art. 73) and is neglected.

Here, the velocity on reaching the pavement is 50 velos; when that velocity ceases, the ball has received 50 velos upwards;—for, 50 velos downwards and 50 velos upwards produce rest;—when the ball is seen moving upwards with 30 velos, it has received 30 velos more; so that in all it has received 80 velos upwards. These velos were produced by a force, which produced acceleration upwards, and which acted for some interval; the interval was very small (too small perhaps to be observed) but still an interval.

The interval given in the question viz. $\frac{1}{100}$th of a second is too small to be actually observed.]

The change in the velocity of the ball is 80 velos; therefore the mass-velocity produced is $\frac{4}{16} \times 80$ pound-velos;
that is, 20 pound-velos,
and this requires an impulse of 20 pulses.

Let x be the *average* number of poundals in the stress; then,
x poundals acting on $\frac{1}{4}$ lb. produce $4x$ celos,

$4x$ celos in $\frac{1}{100}$ secs. produce $\frac{4x}{100}$ pound-velos;

hence, $$\frac{4x}{100} = 20;$$

or, $$x = 500.$$

The *average* force on the ball is 500 poundals;
that is, the weight of $\frac{500}{32}$ pounds; that is, of $15\frac{5}{8}$ lbs.

EXAMPLES. XVIII.

1. A shot of 8 cwt. leaves a fixed 40 ton gun with a muzzle velocity of 2000 ft. per second; find the impulse which has acted upon it. Also if the shot moves over 20 ft. when in the gun, find the resultant uniform push which has acted on the base of the shot.

2. Find how long the shot in question **1** was in motion in the tube of the gun.

3. On Friday Feb. 11th, 1886 at Woolwich Arsenal a gun of 111 tons was fired for the first time with an iron shot of 1800 lbs.; the muzzle velocity was 1600 velos; find the impulse which acted on the shot. Also if the shot moved over 34 ft. while in the gun find the resultant uniform push which was applied to its base to produce this velocity.

4. Find how long the shot in Question **3** was in motion in the tube of the gun.

5. A glass ball of 1 lb. reaches a horizontal pavement with 20 velos and leaves it with 12 velos both velocities being vertical; what impulse has acted on the ball? and supposing the ball and pavement were in contact for $\frac{1}{10}$ sec., find the average external force acting on the ball.

6. A glass ball of 8 oz. is let fall from a height of 16 ft. and is observed after rebounding to rise to the height of 4 ft., find the impulse.

7. Two equal railway carriages of 3 tons moving in opposite directions each with a velocity of 10 miles per hour meet, and rebound each with a velocity of 5 miles an hour; find the impulse which has acted on each. Find also in tons weight the force between the buffers, supposed uniform, assuming that they were in contact for a second.

8. An india-rubber ball weighing 1 lb. strikes a pavement at right angles with a velocity of 30 velos, and rebounds with a velocity of 25 velos; find the impulse which has acted upon it; and find what force acting for $\frac{1}{10}$th of a second would have the same effect.

9. A steam hammer of 10 tons is let fall from the height of 4 ft., and strikes a mass of red-hot iron; supposing that the hammer on touching the iron comes to rest in $\frac{1}{32}$nd of a second, and that the force it exerts is uniform, find the push, and by how much it flattens the red hot iron.

10. A hammer of 2 lbs. hits a nail with a velocity of 40 ft. per second and drives the nail in half an inch; find (i) the force which is exerted on the nail supposing it uniform, (ii) the duration of the impulse.

11. An iron weight of half a ton is let fall from a height of 16 ft. vertically above the head of a pile; the pile is driven into the ground a distance of 1 ft. by the impulse; find (i) the average force, (ii) the duration of the impulse.

*IMPACT.

65. When one mass meets another mass having a different velocity the masses are said to **impinge**.

The meeting of the two masses is called an **impact**.

The impact is said to be **direct** when the masses are moving in the same straight line, provided the direction of the stress caused by the impact is also in that straight line.

Let us consider the case of one mass having a certain velocity impinging on another mass having a different velocity, the masses being each composed of a hard substance, such as a metal or glass.

In one or both of the masses there must be a very rapid change of velocity; that is, there must be a very great acceleration; consequently there must be an immense *mutual pressure* between the masses.

In such cases the impact must certainly occupy some interval of time: the particles of each mass yield slightly in the neighbourhood of the great pressure, and then, either return more or less exactly to their original position (in which case the material is said to be **elastic**), or do not return (in which case the impact causes a permanent indentation, and the material is said to be **inelastic**).

66. It should be noticed [Art. 62] that the smaller the *interval* occupied by a given impulse, the larger is the average *force* which produces that impulse.

Also, the smaller the *interval*, the smaller is the *distance* passed over by the mass during the impulse.

For, let m lbs. moving with u velos have mv pulses applied to it in t seconds; then, assuming that the v velos are produced uniformly, $s=(u+\frac{1}{2}v)\times t$; so that the smaller t, the smaller is s.

67. An impact between two masses causes a stress to be set up between them; which stress lasts for a small interval. The action and reaction of this stress produce *equal* external impulses which act on the two masses in opposite directions.

68. These equal and opposite impulses produce equal and opposite mass-velocities, one in one mass, the other in the other. Hence, the algebraic sum of the *additional* mass velocities produced during an impact is zero; in other words, **when two masses impinge their total mass-velocity is unaltered by the impact.**

69. Two masses M and M' are said to be **inelastic** when after direct impact they move each with the same velocity.

NOTE. When one mass **sticks** to another on impact; or when one mass **penetrates** another, as a bullet penetrates a piece of wood, and stays in it; in fact in every case in which the masses have the same velocity after impact, they are said to be *inelastic.*

Example i. *A mass* M *of* 3 lbs. *having* 5 velos *impinges directly on a mass* M′ *of* 7 lbs. *at rest; find their subsequent motion, the masses being* inelastic.

Since the masses are inelastic, their velocities are made equal by the impact; let their velocity after impact be v velos.

$$\text{Before impact}\begin{cases}\text{the mass-velocity of } M \text{ is } 3\times 5 \text{ pound-velos.}\\ \text{the mass-velocity of } M' \text{ is } 0.\end{cases}$$

$$\text{After impact}\begin{cases}\text{the mass-velocity of } M \text{ is } 3\times v \text{ pound-velos.}\\ \text{the mass-velocity of } M' \text{ is } 7\times v \text{ pound-velos.}\end{cases}$$

Hence, by Art. 68, $3v+7v=15$;

or, $v=\frac{3}{2}$;

that is, after impact they move together with a velocity of 1½ ft. per second.

Example ii. *A leaden ball of* 1 lb. *impinges directly with a velocity of* 1000 ft. per sec. *on a mass of* 1 cwt. *moving in the same direction with a velocity of* 20 ft. per sec; *find their subsequent velocity.*

Let them have v velos after the impact; then, since the total mass-velocity is unchanged

$$1\times 1000+112\times 20=v\times(112+1),$$

or,
$$v=\frac{3240}{113}$$
$$=28\cdot 6\ldots$$

That is, the masses move together with a velocity of 28·6... ft. per second

EXAMPLES. XIX.

The following impacts are all direct.

1. An inelastic ball of 1 cwt. moving with a velocity of 20 ft. per sec. impinges against an equal ball at rest; find their subsequent velocity.

2. An inelastic particle of m lbs. having v velos impinges directly on another of m' lbs. at rest; find their subsequent velocity.

3. An inelastic particle of 20 lbs. meets another of 2 lbs.; each particle has 11 velos but in opposite directions; find their subsequent motion.

4. A railway carriage of 10 tons is moving towards a train of 100 tons with a velocity of 4 miles an hour; at the instant of collision the carriage is coupled to the train; find the subsequent velocity.

5. A particle of 1 cwt. meets another of 28 lbs. having a velocity of 15 miles an hour and after the impact they both are at rest; with what velocity was the 1 cwt. moving?

6. A particle of 1 lb. moving with 30 velos overtakes another particle of 2 lbs. and after impact they move together with 25 velos; what was the velocity of the 2 lbs. before impact?

7. An inelastic mass impinges on another of twice its mass at rest; shew that the impinging body loses two-thirds of its velocity by the impact.

8. An inelastic particle of mass m and velocity v impinges directly on a particle of mass m'; they are at rest after the collision; shew that the velocity of the second particle was $-\frac{m}{m'}v$.

9. Three equal inelastic balls are placed in a line, not in contact; an equal ball moving in the same line with 20 velos impinges on the first, these two impinge on the second and then the three on the third; find their joint velocity after impact.

10. An engine of m tons impinges with a velocity of 4 miles an hour on each of 4 inelastic trucks each of $\frac{1}{20}m$ tons placed on the same line of rails and separated by a small interval; find the velocity of the train after impact.

11. A rifle bullet of $\frac{1}{2}$ oz. having 1200 velos impinges on a block of wood weighing 5 cwt. at rest and free to move in the direction of the impact; what is the velocity of the block immediately after impact.

12. A particle is let fall from the top of a tower and at the same instant an equal particle is thrown vertically upwards from the foot of the tower with a just sufficient velocity to carry the particle under the action of gravity to the top of the tower. The two particles being inelastic impinge; shew that directly after impact the particles are at rest; and if the tower is 128 ft. high find how long the first particle takes to reach the ground.

* ELASTICITY.

70. DEF. Two masses are said to be **elastic** when after impact, their velocities in the direction of the impulse are not the same.

When two elastic masses impinge they probably behave much as inelastic masses up to the time when they are moving with the same velocity; the property of elasticity seems to cause a continuation of the impulse so that the masses *recoil* from each other. The amount of recoil must be in some degree proportional to the magnitude of the impulse and will depend on the nature of the material of the masses.

71. Newton observed, that when two spheres of given substance impinge directly on each other, the relative velocity after impact is e times the relative velocity before impact and in the opposite direction; where e is a number not greater than 1. This number e depends on the *nature* of the substances of the two spheres, and is called their **coefficient of restitution**; hence we say that

72. When two elastic masses m lbs. and m' lbs. impinge directly on each other, u velos and u' velos being their velocities before impact, v velos and v' velos their velocities after impact, then

$$v - v' = -e(u - u');$$

where e is a number, not greater than 1, which depends on the materials of the masses and is called the *coefficient of restitution.*

In other words, the two masses separate after impact with a velocity which is e times that velocity with which they were approaching each other before impact.

Velocity of separation $= e$ times velocity of approach.

When the coefficient of restitution of a substance $= 1$ the substance is said to be **perfectly elastic.**

Example i. *Two masses of* 5 lbs. *and* 20 lbs. *having velocities of* 20 velos *and* − 10 velos, *whose* coefficient of restitution *is* ⅔, *impinge; find their subsequent velocities.*

Let v velos and v' velos respectively be their subsequent velocities.

Then since their total mass-velocity is unaltered by the impact

$$5\times 20-20\times 10=5v+20v' \qquad \text{(i)}$$

also, by Art. 41, $$v-v'=-\tfrac{2}{3}(20+10) \qquad \text{(ii)}$$

whence $$5v+20v'=-100,$$

and $$v-v'=-20;$$

$$\therefore\ 25v=-100-400;\ \text{or, } v=-20;\ \text{also, } v'=0.$$

Hence, after the impact the 5 lbs. moves in the negative direction with velocity 20 ft. per second, and the 20 lbs. comes to rest.

Example ii. *Two masses of* m lbs. *and* m′ lbs. *impinge with velocities* u velos *and* u′ velos; *find the condition that they may interchange velocities.*

Here u' velos and u velos are to be their respective velocities after impact;

hence, $$mu+m'u'=mu'+m'u \qquad \text{(i)}$$

or, $$(m-m')(u-u')=0,$$

whence, $$m=m'$$

(for, since the masses impinge, u cannot $=u'$).

Again, $$u-u'=-e(u'-u) \qquad \text{(ii)}$$

or, $$(u-u')(1-e)=0,$$

whence, $$e=1.$$

Thus, in order that two masses after impact may interchange velocities they must be of equal mass and perfectly elastic.

EXAMPLES. XX.

The coefficient of restitution is here denoted by e.

1. A sphere of 6 lbs. having 20 velos overtakes another of 4 lbs. having 12 velos; determine their velocities after impact, in the case in which $e=\frac{1}{2}$.

2. A particle of 28 lbs. having 8 velos impinges directly on another of 14 lbs. having − 16 velos; determine their subsequent velocities when $e=\frac{1}{3}$.

3. Two elastic balls ($e = \frac{3}{4}$) of masses $3m$ and m and velocities $3v$ and $-5v$ impinge; find their subsequent velocities.

4. A ball of $3m$ impinges on another m at rest which afterwards moves with 10 velos, e being $\frac{2}{3}$; find the velocity of the first ball before impact.

5. The result of a direct impact between two spheres of elasticity $\frac{1}{4}$, one of which is at rest, is that one of them has after impact twice the velocity of the other; prove that one ball has $\frac{2}{3}$ the mass of the other.

6. A series of equal elastic balls of elasticity e are placed in a line separated from each other by short distances; another ball of equal mass moving in the same line impinges on the first with velocity v; find the velocities after impact of the first, second, third and nth balls.

7. A ball of mass m and elasticity e is projected vertically under the action of gravity with 64 velos; another ball of mass m' is simultaneously let fall from a height of 64 ft. vertically above m; find when and where they impinge, and their velocities after the impact.

8. Two elastic spheres impinge directly with equal velocities; find the ratio of their masses that one of them may be reduced to rest by the impact.

9. A shot of 1 cwt. is projected from a 20 ton gun placed on a smooth horizontal plane, with a horizontal muzzle velocity of 3000 velos; find the velocity of the recoil of the gun.

10. Two railway carriages B and C of m lbs. and m' lbs. stand on the same line of perfectly smooth rails separated by a short distance; a third carriage A of m lbs. impinges on B and then consequently B impinges on C; prove that A will impinge a second time on B if m' is greater than $\frac{2em}{1+e^2}$, where e is the coefficient of restitution.

11. Two equal spheres A, B are connected by a string and laid on a smooth horizontal table, at a distance from each other which is less than the length (l ft.) of the string; a velocity u is given to A in the direction BA; on the string becoming tightened a direct impulsive tension is set up; if the coefficient of restitution of the string be e, and of the spheres themselves e', find the velocity of the spheres (i) after the first impulse of the tension, (ii) after the first impact of the spheres.

12. A series of equal masses of 1 lb. each are connected by light inelastic strings each 4 ft. long. They are placed in a straight row touching each other on a smooth horizontal plane; to the first mass is applied a force of 2 poundals in the direction of the row, and when the first mass has moved over 4 ft., the string applies a direct impulse to the second mass; and when the first two have moved over 4 ft. more, the string applies a direct impulse to the third mass, and so on; find the initial velocities of the second, third and fourth masses.

Example iii. *A small particle of mass* m lbs. *moving with* u velos *impinges directly on a large mass of* M lbs. *at rest; find their subsequent motion, the coefficient of restitution being* e.

We have, with the usual notation,

$$mu = mv + Mv', \qquad \text{(i)},$$

$$v - v' = -eu; \qquad \text{(ii)},$$

whence, $$v = \frac{m - eM}{m + M}u; \text{ and, } v' = \frac{m}{M + m}(1 + e)u.$$

Now suppose M to be so much larger than m, that the ratio $\frac{m}{M}$ is too small to be observed, and may be considered to be nothing; then

$$v' = 0; \text{ and, } \qquad v = \frac{\frac{m}{M} - e}{\frac{m}{M} + 1}u = -eu.$$

This is the case when a ball of mass m lbs. comes into collision with the Earth of mass M lbs. Hence we say that

73. When an elastic ball impinges against any surface fixed to the Earth, its velocity after impact is $-e$ times that before impact.

Example. A particle let fall from a height of 16 ft., *impinges on a horizontal pavement, the coefficient of restitution being* e; *it again falls and rebounds; and so on; find when the particle will come to rest.*

The particle has an acceleration 32 celos downwards.

Let its velocity on striking the ground be v velos, and on reaching the ground after each rebound, v_1 velos, v_2 velos, etc.; then we have

$$v^2 = 2as = 2 \times 32 \times 16; \quad \text{or, } v = 32;$$

$$v_1 = ev; \quad v_2 = ev_1 = e^2v; \quad v_3 = ev_2 = e^3v; \text{ and so on.}$$

To ascertain the intervals, we observe that to produce 32 velos, 32 celos require 1 sec.; to produce $e \times 32$ velos, 32 celos require e times 1 sec., and so on;

In the successive bounds ev velos, e^2v velos,... are in turn first destroyed and then reproduced by the acceleration due to gravity.

Hence the sum of the intervals occupied by the bounds are

$$\{2e + 2e^2 + 2e^3 + \ldots\ldots\ldots\} \text{ seconds.}$$

The limit of this series is $\frac{2e}{1-e}$ seconds.

Therefore the particle comes to rest in

$1+\frac{2e}{1-e}$, or, $\frac{1+e}{1-e}$ seconds, from the instant at which it was let fall.

EXAMPLES. XXI.

The coefficient of restitution of an elastic ball is sometimes called its elasticity.

1. A particle let fall from a height of 16 feet, impinges on a horizontal pavement; find the interval occupied by the first rebound, the coefficient of restitution being $\frac{1}{2}$.

2. A particle let fall from a height of 48 ft., impinges on a horizontal pavement; find the elasticity that in its first rebound it may rise to a height of 32 ft.

3. A particle let fall from a height of h ft., impinges on a horizontal pavement, the coefficient of restitution being e; find

(i) the height of the first rebound,
(ii) the interval between the first two impacts,
(iii) the time in which the particle comes to rest.

4. An elastic ball falls from a height h ft. to a horizontal plane and then rebounds; falls again and rebounds; and so on; find the sum of the vertical distances passed over.

5. Supposing that in a direct impact between two particles of elasticity e, we consider the impulse between the particles to be divided into two parts, the first part being that which causes the two particles to be relatively at rest; shew from Art 71 that these two parts of the impulse are in the ratio of 1 to e.

6. The 110 ton gun projects its shot of 1800 lbs. with a muzzle velocity of 2100 velos; what will be the initial velocity of the recoil of the gun, supposing the shot projected horizontally, and the gun to be placed on a smooth horizontal plane?

7. Two masses m lbs. and m' lbs. of elasticity e impinge directly with u velos and u' velos respectively; prove that if v velos and v' velos are their velocities after impact, then

$$v=\frac{(m-em')\,u+m'(1+e)\,u'}{m+m'}$$

and

$$v'=\frac{(m'-em)\,u'+m\,(1+e)\,u}{m+m'}.$$

*CHAPTER V.

CHANGE OF UNITS.

74. Our units have been, *a foot*, *a second*, *a pound*; from which are derived the units, *a velo*, *a celo*, *a poundal*, *a pulse*.

The velocity chosen for the *unit velocity* is always that of a point which passes uniformly over the unit distance in the unit interval.

The acceleration chosen for the *unit acceleration* is always that of a point whose velocity is increased uniformly by the unit velocity in the unit interval.

The force chosen for the *unit force* is that which when acting on the unit mass produces in it unit acceleration.

75. We give some examples involving a change of units.

Example i. *When the* unit acceleration *is* 32 feet per sec. per sec. *and the* unit velocity *is* 10 feet per sec., *find the* unit distance *and the* unit interval.

Let the unit interval be x seconds,

let the unit distance be y feet; then,

the unit velocity is y ft. per x seconds;

or, $\frac{y}{x}$ ft. per sec.;

hence, $$10=\frac{y}{x}.$$

The unit acceleration is 10 velos per x seconds;

or, $\frac{10}{x}$ velos per second;

hence, $$32=\frac{10}{x}; \qquad \text{or, } x=\tfrac{5}{16}.$$

Also, $$x=\frac{y}{10}=\tfrac{5}{160}=\tfrac{1}{32}.$$

Thus the unit distance is $\frac{5}{16}$ ft. the unit interval $\frac{1}{32}$ secs.

Example ii. *When the area of a field of* 10 acres *is the* unit area, *and the* acceleration due to gravity *the* unit acceleration, *find the* unit interval.

A field of 10 acres contains 48400 sq. yds.,

that is, $(220)^2$ sq. yds.;

hence, the unit distance is 220 yds., or 660 ft.

Let the unit interval be x seconds,

The unit velocity is 660 ft. per x seconds;

that is, $$\frac{660}{x} \text{ velos.}$$

The unit acceleration is $\frac{660}{x}$ velos per x seconds;

that is, $$\frac{660}{x^2} \text{ celos.}$$

But the unit acceleration is g celos, or 32 celos,

therefore, $$32=\frac{660}{x^2},$$

or, $$x^2=\frac{165}{8}=20\cdot625,$$

whence, $$x=4\cdot54\ldots$$

The required unit is about 4½ secs.

Example iii. Given, *that the* unit of acceleration *is* a celos, *the* unit of velocity, u velos; *find the* units of time and distance.

Let the unit interval be x secs.

let the unit distance be y ft.

Then the unit velocity is y ft. per x secs.

or, $$\frac{y}{x} \text{ velos,}$$

and the unit acceleration is $\frac{y}{x}$ velos per x secs.

or, $$\frac{y}{x^2} \text{ celos.}$$

Hence $$a=\frac{y}{x^2}, \quad \text{and } u=\frac{y}{x}$$

or, $$x=\frac{u}{a}, \quad y=\frac{u^2}{a}.$$

Thus, the unit interval is $\frac{u}{a}$ secs., the unit distance is $\frac{u^2}{a}$ ft.

EXAMPLES. XXII.

1. The unit distance being a yard and the unit interval a minute, express the unit velocity in velos and the unit acceleration in celos.

2. The unit distance being a mile and the unit interval an hour, express the unit velocity in velos and the unit acceleration in celos.

3. The unit distance being a yds. and the unit interval b seconds, express the unit velocity in velos and the unit acceleration in celos.

4. The unit velocity being 60 miles an hour, and the unit distance 1 yd., find the unit interval.

5. The unit velocity being 100 yds. per minute and the unit interval 5 secs., find the unit distance.

6. The unit acceleration being the double of that due to gravity and the unit distance 1 mile, find the unit velocity.

7. The unit velocity being 40 ft. in 11 secs. and the unit acceleration 3 yds. per minute per minute, find the units of time and distance.

8. The unit mass being 1 cwt. and the unit force the weight of 7 lbs., the unit velocity 10 velos, find the unit distance.

9. The unit force being the weight of 1 lb., and the unit interval and unit distance being 1 sec. and 1 ft. respectively, find the unit mass.

10. The unit mass being 1 ton, the units of interval and of distance an hour and a mile, find the unit force.

11. Find the measure of the acceleration due to gravity when the unit distance is a metre, say 3·28 ft., and the unit interval a second.

12. If f is the measure of an acceleration when m secs. and n ft. are the unit interval and unit distance respectively, express the acceleration in celos.

13. If f and F be the measures of the same acceleration when the unit interval and unit time are t secs. s ft. and τ secs. and σ ft., respectively, prove that $F=\frac{s}{t^2}\times\frac{\tau^2}{\sigma}\times f.$

14. Taking the centimetre ($\frac{1}{100}$ of a metre = ·0328...ft.) as the unit distance, the second as the unit interval and the gram (= ·0022... lbs.) as the unit mass, find the unit velocity in velos, the unit acceleration in celos, and the unit force in poundals.

N.B. This system of units is called the C.G.S. [**Centimetre, Gramme, Second**] system, and the unit force is called a **dyne**.

NOTE. We may remark that in Statics the unit force generally chosen is *the weight* of 1 lb.; in this case, since we ought to take as the unit-mass the mass in which unit force produces unit acceleration, our unit mass must be g lbs.; the disadvantage of this is that g has different values for different latitudes.

SECTION II.

MOTION IN ONE PLANE.

CHAPTER VI.

DIRECTION.

76. Distance has **direction.**

We have hitherto supposed all our distances to be measured in the same direction, (or in exactly opposite directions), so that it was only necessary to consider the *magnitude* and *sign* of our distances.

In what follows we shall always consider the *direction* of a distance to be an essential part of it. As a necessary consequence we must consider *direction* to be an essential part also of all quantities which vary directly as distance, such as *velocity*, *acceleration*, *force*, *impulse*.

77. Quantities of which direction is an essential part, must be carefully distinguished from those quantities in which it is not.

Quantities having direction are called **vector** quantities.

Distance, *velocity*, *acceleration*, *force*, *impulse* have direction.

Quantities without direction are called **scalar** quantities. *Time*, *mass*, *volume*, *speed* have no direction.

78. It is convenient to use the word **speed** to denote the *magnitude* only of a velocity; just as we use the word *length* to denote the *magnitude only* of a distance. The **average speed** of a point moving in a *curved* line is that which varies directly as the *length* of the curved line passed over by the point, and inversely as the interval occupied.

The average speed of a point moving in a *straight* line is the same as its average velocity.

When we speak of a train moving at the rate of 40 miles an hour for a certain interval, we mean that its *average speed* is 40 miles per hour.

A point may have a constant speed while its velocity is changing (in direction).

Similarly we may use the word **quickening** to denote the *magnitude only* of an acceleration.

79. PROP. Any quantity which has direction and magnitude may be represented by a finite straight line with an arrow-head.

For the direction of the quantity can be represented by the *direction* of the line with the arrow-head, and the numerical measure of the quantity can be indicated by that of the *length* of the line.

When a distance is indicated by the two letters at its extremities, the direction of the arrow-head may be indicated by *the order of the letters.*

80. A distance can always be found which represents the *combined effect*, or the *resultant*, *of two given distances in different directions.*

The combined effect of, (or the result of adding together) two non-directional, (or scalar) quantities is the arithmetical sum of their measures.

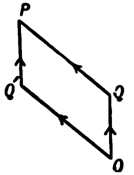
The same is true of two directional, (or vector) quantities which have the same direction.

But the combined effect of two directional, (or vector) quantities having different directions requires definition.

Example. A point P is said to have a distance of 2 inches northwards from a given point O, and *also* a distance of 3 in. towards the north-west from O, when the position of P is found as follows:

Starting from O measure OQ 2 in. towards the north; then *from* Q measure QP 2 in. towards the north-west.

Or thus; starting from O measure OQ' 3 in. towards the north-west and then *from* Q' measure $Q'P$ 2 in. towards the north.



It will be seen that the resulting position of P is the same in each case; for OQ, OQ' are sides of a parallelogram and P is at the corner opposite to O.

81. We may illustrate the meaning of a point having two simultaneous distances from a starting point thus:—

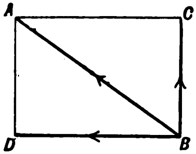
Suppose a sheet of glass to be placed on paper and the point P to be a moveable mark placed on the *glass*.

Let the mark be moved a distance of 3 in. on the glass towards the north-west, and let the glass be moved relatively to the paper, a distance of 2 in. northwards.

It will be seen that the resulting position of the point P with reference to the paper is that found as in the Example in Art. 80.

And it makes no difference to the ultimate position of P whether these distances are passed over *simultaneously*, or whether they are passed over *separately*, one of the movements being finished before the other is made.

Example i. *A point* A *is distant* 3 miles *towards the* North *and* 4 miles *towards the* West *from a point* B; *what is the* total distance *of* A from B?



Draw BC northwards 3 units of length; draw BD westwards 4 units of length; complete the parallelogram $CBDA$;

then $CBDA$ is a right-angled parallelogram

and $$AB^2 = AD^2 + BD^2.$$

Hence if AB contains x units of length

$$x^2 = 3^2 + 4^2 = 9 + 16 = 25;$$

therefore $$x = 5.$$

Thus the distance AB is 5 units of length.

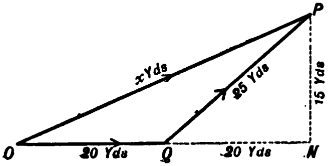
Hence it follows that the total distance of the two places described in the question from each other is 5 miles.

And the direction is such that if AB make an angle α with BD,

then, $\tan \alpha = \frac{DA}{BD} = \frac{3}{4}$.

Example ii. *A man walks* 20 yards eastwards *and then* 25 yards *in a direction making an* angle whose sine is $\frac{3}{5}$ *with the line pointing eastwards and on the north side of it; find the* direction and magnitude of the resultant distance *of the man from his starting point.*

Let OQ represent a distance of 20 yards eastwards; produce OQ to N; let QP represent the distance of 25 yds., QP being drawn so that $\sin PQN = \frac{3}{5}$.



Draw PN perpendicular to OQ produced; then, $\frac{NP}{QP}$ is the sine of the angle which QP makes with ON; therefore $\frac{NP}{QP} = \frac{3}{5}$.

Let PN contain y yds. Then $y = \frac{3}{5}$ of $25 = 15$.

Similarly, QN may be shewn to contain 20 yds.

Let OP contain x yds. Then since $OP^2 = ON^2 + NP^2$

$$x^2 = (20+20)^2 + (15)^2 = 1600 + 225$$
$$= 1825;$$

therefore,
$$x = \sqrt{(1825)} = 42 \cdot 7 \ldots$$

Hence, the required distance is 42·7 yds., also its direction makes an angle whose tangent is $\frac{15}{40}$, or $\frac{3}{8}$, with the line towards the East.

EXAMPLES. XXIII.

1. A man walks 300 yards northwards and then 400 yards westwards; another man starting from the same place walks 400 yards westwards and then 300 yards northwards; shew that the two men are then at the same total distance from the starting point; and find that distance.

2. A dog on an ice-floe goes 3 miles northwards while the floe itself travels 4 miles eastwards; how far is the dog then from the place on the earth's surface from which he started?

3. A fly crawls along the floor of a railway carriage a distance of 2 ft., at right angles to the line of rails, and at the same time the carriage moves a distance of 10 ft.; find approximately the distance moved over by the fly.

4. A man swims across a river, and in 40 seconds he swims 20 yards; in the same time the current of the river has carried him 15 yards down stream; how far is he from his starting point after 40 secs.?

5. A fly, in a railway carriage which is moving at the rate of 10 ft. per sec. northwards, crawls at the rate of 5 ft. in 10 seconds across the floor of the carriage towards the north-east; what distance will the fly have gone in 10 seconds?

6. On the floor of a railway carriage which is moving with 4 velos northwards, four flies A, B, C, D start from the same place and crawl each with 3 velos, A Northwards, B Southwards, C Westwards, and D towards the North-West. Find their distances from the starting point after 2 seconds.

* *The following examples require the use of the Trigonometric Ratios.*

7. A man walks 350 yards in a certain direction and then turning his direction through an angle of 120 degrees walks 350 yards more; find the direction and magnitude of his resulting distance from his starting point.

8. A man on the deck of a ship moving due north, walks 20 yards towards the north-east while the ship goes 100 yds.; find the direction and magnitude of the whole distance passed over by the man.

9. The distance of a point B from another A is the sum of two equal distances of 20 yds. inclined to each other at an angle of 60 degrees; find the distance of B from A.

10. When the wind is due North a certain boat can sail either towards the north-east or north-west; it sails first 2 miles towards the N.E., then 2 miles towards the N.W., then 1 mile more towards the N.E.; find the magnitude and direction of its resulting distance from its starting point.

11. I walk 3 miles northwards, then 2 miles in a direction pointing 30^0 to the east of north; what will then be my distance from my starting point?

12. I walk 3 miles in a straight line, I then change my direction through an angle whose sine is $\frac{4}{5}$ and walk 5 miles straight on, I then *turn* (in the same direction as before) through a right angle and walk 1 mile; how far shall I then be from the place from which I started?

82. DEF. The **resultant** of two *distances* from a given point is the one *distance* which is the combined effect or sum of the two distances, taking into account their direction as well as their magnitude.

83. PROP. **The parallelogram of Distances.** *The resultant of two given distances* OA, OB *is* OR, *the diagonal of the parallelogram* AOB.

This follows because the opposite sides of a parallelogram are equal.

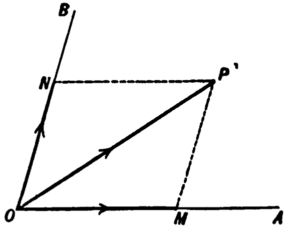
In the figure below the distance OP is the resultant of the distances OM and MP; that is, of OM and ON, for MP is equal and parallel to ON.

84. When the two distances are in the same direction their resultant coincides with the two distances placed end and end, so that its magnitude is their *sum*.

When the two distances are in opposite directions the magnitude of their resultant is their *difference*.

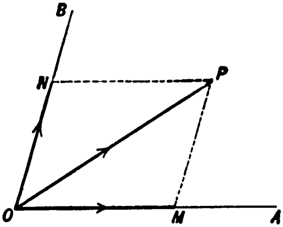
85. It is often convenient to express the distance of one point from another as the resultant of two distances in given directions.

Example. Express the distance OP *as the resultant of two distances in directions parallel to* OA *and* OB *respectively.*



Through P draw PM and PN parallel to OB and OA respectively. Then the distance OP is the resultant of the distances OM and ON.

86. When the one distance OP is expressed as the sum of two others OM and ON, the two distances OM and ON are called the **components** of the distance OP.



The distance OP is said to be **resolved** into its two *components* OM and ON in those directions.

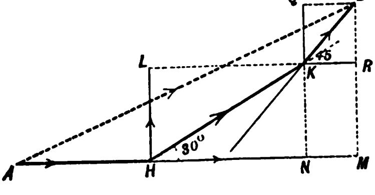
The distance OP is clearly the *resultant* of its two components OM and ON.

It is usually convenient to take the two component directions **at right angles** to each other.

When the two components are mutually at right angles, each component is called **the** *resolved part* of the resultant in its own direction.

Example i. *The road from* A to B *runs in a straight line for* 2 miles *from* A *to* H; *then after turning through an angle of* 30°, *it runs in a straight line for* 3 miles *from* H *to* K; *then after turning through an angle of* 15°, *runs in a straight line for* 1 mile *from* K to B; *how far is it from* A *to* B *as the crow flies?*

Resolve the distance HK into its two resolved parts HN and $HL = NK$ along and perpendicular to AH.



Resolve the distance KB into its two resolved parts $KR=NM$ and $KQ=RB$ along and perpendicular to AH.

Then,

$$HN=HK\cos 30=\tfrac{1}{2}\sqrt{3}\times 3 \text{ miles},$$

$$NK=HK\sin 30=\tfrac{1}{2}\times 3 \text{ miles},$$

$$NM=KR=KB\cos 45^0=\frac{1}{\sqrt{2}}\times 1 \text{ mile},$$

$$RB=\quad KB\sin 45^0=\frac{1}{\sqrt{2}}\times 1 \text{ mile}.$$

Hence, the number of miles in AM is

$$2+\tfrac{1}{2}\times 3\sqrt{3}+\tfrac{1}{2}\sqrt{2}.$$

The number of miles in MB is

$$\tfrac{1}{2}\times 3+\tfrac{1}{2}\sqrt{2}.$$

Therefore, the number of miles in AB is

$$\begin{aligned}&\sqrt{\{(2+\tfrac{1}{2}\times 3\sqrt{3}+\tfrac{1}{2}\sqrt{2})^2+(\tfrac{3}{2}+\tfrac{1}{2}\sqrt{2})^2\}}\\ &=\sqrt{\{(2+\tfrac{1}{2}\times 3\times 1{\cdot}7320\ldots+\tfrac{1}{2}\times 1{\cdot}4142\ldots)^2+(1{\cdot}5+\tfrac{1}{2}\times 1{\cdot}4142\ldots)^2\}}\\ &=\sqrt{\{(5{\cdot}305\ldots)^2+(2{\cdot}207\ldots)^2\}}=\underline{5{\cdot}746\ldots\text{ nearly}}.\end{aligned}$$

Example ii. *The distance of* B *from* A *is the* resultant *of the three distances* AP, AP′, AP″ *of* 2 miles, 3 miles and 1 mile *respectively where the* angle PAP′ = 30^0, *and the* angle P′AP″ = 15^0; *what is the* distance of B from A?

It will be found that this is only another way of stating *Example* i.

EXAMPLES. XXIV.

1. Resolve a distance of 100 yards into two components mutually at right angles one of which makes an angle of 30^0 with it.

2. Resolve a distance of 25 yards into two components mutually at right angles one of which makes an angle whose sine is $\frac{3}{5}$ with it.

3. A man walks 200 yards in a straight line, then turns through an angle of 45^0 and walks 150 yards further in a straight line; find his resultant distance from the starting point.

4. Find the resultant of two distances of 300 yards and 450 yards making an angle of 45^0 with each other.

5. I walk 1 mile towards the north-east; what is the *resolved part* of this distance in the northerly direction? what is the *resolved part* of this distance towards the north-west?

6. Find the resolved part of a distance of 360 yards in the direction which makes an angle of 60^0 with it.

87. When a point has a given uniform velocity u velos in a given direction, then after an interval t seconds the point is at a certain *distance* from its initial position; this distance is ut feet, and is in the direction of the velocity.

This distance is called **the distance due to the velocity**.

88. A point is said to have **simultaneous** velocities and accelerations when its distance after an interval from its initial position is the resultant of the distances due to those velocities and accelerations separately.

Example. A point has vertically − 32 celos, *and* 200 velos, *and it has* 40 velos *in a horizontal direction; find its position after* 3 seconds.

Let O be its initial position.

Draw ON horizontally to represent 3 × 40 ft.

From N draw NQ vertically upwards to represent 3 × 200 ft.
and from Q draw QP vertically downwards to represent $\frac{1}{2} \times 32 \times 9$ ft.

Then P represents the position of the point after 5 seconds.

EXAMPLES. XXV.

Find the position of a point having the following;

1. 3 velos northward and 4 velos eastwards after 30 seconds.

2. 2 velos southwards and 3 velos eastwards after 15 secs.

3. 4 velos northwards and 7 velos southwards after 8 secs.

4. 20 velos northwards, 15 velos eastwards and 15 velos northwards after 8 secs.

5. 20 velos and − 3 celos westwards after 10 seconds.

6. Three velocities of 20 velos each making an angle of 120° with each other, after 10 seconds.

7. 20 velos northwards and 32 celos westwards after 2 secs.

8. Three accelerations, 15 celos westwards, 20 celos northwards *and 15 celos* eastwards after 1 second.

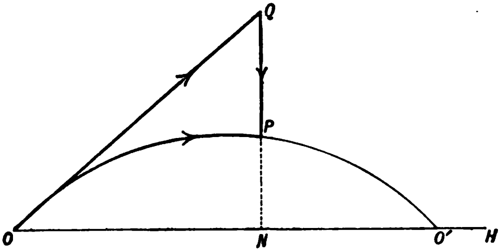
89. The motion of a projectile affords an important example of a point which after any interval has two simultaneous distances from its initial position.

Example. A particle is projected with u velos *in a direction making an* angle α *with the horizontal plane; find its position after* t seconds.

The only force acting on the particle is its own weight, which acts vertically downwards.

By Art. 33 this force produces in the mass the same acceleration whether the particle has other velocity or not.

Hence, the distance of the particle after t seconds from its initial position is the resultant of two distances, one due to the u velos, the other due to the acceleration g celos vertically downwards.



Let O be the initial position of the particle P;

let OH be a horizontal line;

then we may find the position of P after t seconds thus:

Drawing OQ, making an angle α with OH, so that OQ represents a distance of ut feet;

from Q draw QP vertically downwards, so that QP represents $\frac{1}{2}gt^2$ feet.

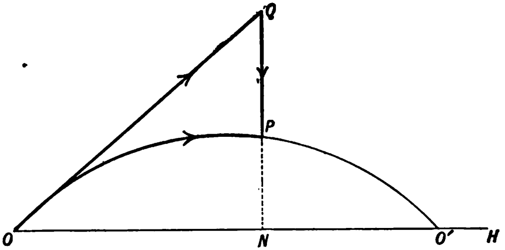
Then P is the position of the particle after t seconds.

For OQ is the distance due to the velocity u velos, and QP is the distance due to the acceleration g celos.

90. DEF. When a particle is projected with a given velocity under the action of its own weight, its **range** is the horizontal distance from its initial position at which the particle strikes the ground;
its **time of flight** is the interval occupied between its start and the instant at which it strikes the ground.

91. PROP. *To find* (i) *the* range, (ii) *the* time of flight *and* (iii) *the* greatest height *of a particle projected from a point on a horizontal plane.*

Let u velos be the initial velocity; α the angle of projection with the horizontal plane.



Draw $OQ = ut$ feet, making an angle $HOQ = \alpha$, with the horizontal line OH; draw $QP = \frac{1}{2}gt^2$ feet vertically downwards; then P is the position of the particle after t seconds.

Produce QP to cut OH in N.

Then, $ON = OQ \cos \alpha = ut \cos \alpha$ feet.

Also $NP = NQ - PQ = OQ \sin \alpha - PQ$

$= (ut \sin \alpha - \frac{1}{2}gt^2)$ feet.

When P strikes the ground NP vanishes;
hence, if t seconds be the time of flight,

$$ut \sin \alpha - \tfrac{1}{2}gt^2 = 0,$$

that is, either $t = 0$, or, $t = \dfrac{2u \sin \alpha}{g}$.

Hence, when P reaches the ground *again* at O'

$$t = \frac{2u \sin \alpha}{g},$$

And then, OO', [which $= (ut \cos \alpha)$ ft.] $= \dfrac{2u \sin \alpha}{g} \times u \cos \alpha$ ft.

Hence the range is $\dfrac{2u^2 \sin \alpha \cos \alpha}{g}$ feet, (i)

and the time of flight is $\dfrac{2u \sin \alpha}{g}$ seconds. (ii)

Also, after t seconds $NP = (ut \sin \alpha - \frac{1}{2} g t^2)$ feet

$$= \left[\frac{1}{2} \frac{u^2 \sin^2 \alpha}{g} - \left(\frac{u \sin \alpha}{g} - t\right)^2 \frac{g}{2}\right] \text{ feet;}$$

this is *greatest* when $t = \dfrac{u \sin \alpha}{g}$; that is, at the *middle* of the time of flight; and then $NP = \frac{1}{2} \dfrac{u^2 \sin^2 \alpha}{g}$ feet, which is the greatest height. (iii)

Example. A projectile has initially 1600 velos *in a direction making an angle* whose sine is $\frac{1}{10}$ *with a horizontal plane; find the* time of flight, *the* range, *and the* greatest height.

Here, $\sin \alpha = \frac{1}{10}$

and $\cos \alpha = \sqrt{(1 - \sin^2 \alpha)} = \sqrt{\frac{99}{100}} = \frac{1}{10}\sqrt{99}$

$= {\cdot}9949\ldots$

The vertical distance after t seconds is

$$(ut \sin \alpha - \tfrac{1}{2} g t^2) \text{ feet;}$$

that is, $(1600 \times \frac{1}{10} \times t - 16t^2)$ feet.

This is zero when $t = 10$.

Hence, the time of flight is 10 seconds (i).

The horizontal distance after t seconds is $ut \cos \alpha$ feet.

Hence, the range is $1600 \times 10 \times {\cdot}9949\ldots$ that is, about 15918 feet (ii).

The greatest height is the vertical distance at the middle of the time of flight;

that is, $(1600 \times 5 \times \frac{1}{10} - 16 \times 25)$ feet.

Hence, the greatest height is 400 feet (iii).

EXAMPLES. XXVI.

Find the range, the time of flight and the greatest height when a particle is projected from a point on a horizontal plane under the action only of its weight; the direction of projection making the following angles with the horizon and the initial velocity being as follows.

1. Angle, 45^0; velocity, 100 velos.

2. Angle, 45^0; velocity, 50 velos.

3. Angle, 45^0; velocity, 64 velos.

4. Angle, 30^0; velocity, 250 velos.

5. Angle, 60^0; velocity, 100 velos.

6. Angle, 60^0; velocity, 50 velos.

7. The angle whose sine is $\frac{3}{5}$; velocity 75 velos.

8. The angle whose sine is $\frac{4}{5}$; velocity 125 velos.

9. The angle whose sine is $\frac{1}{100}$; velocity 1000 velos.

10. The Martini-Henry rifle gives its bullet a muzzle velocity of 1,315 velos; find its range when it is elevated to an angle whose sine is $\frac{1}{120}$, neglecting the resistance of the air.

11. The new Enfield-Martini rifle gives its bullet a muzzle-velocity of 1570 velos; find its range when it is elevated to an angle whose sine is $\frac{1}{130}$, neglecting the resistance of the air.

12. Find the height of the bullet in Questions **7** and **8** at the middle of the flight.

13. The 110 ton gun gives its shot of 1800 lbs. (when using 950 lbs. of Waltham Powder) a muzzle velocity of 2100 velos; find its range on a horizontal plane when elevated to an angle whose sine is $\frac{1}{10}$.

14. The 110 ton gun gives a muzzle velocity of 2128 velos when using 1000 lbs. of powder; find the range on a horizontal plane when the elevation is an angle whose sine is $\frac{1}{5}$.

CHAPTER VII.

THE PARALLELOGRAM OF VELOCITIES.

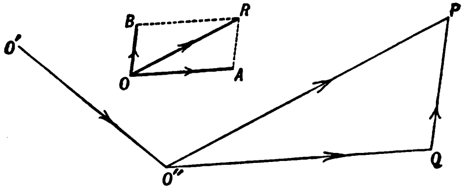
92. DEF. The **resultant** of two simultaneous *velocities* is a *velocity* such that the distance due to it after any interval is equal to the resultant of the two distances due to the two simultaneous velocities.

93. The two simultaneous velocities are called the *components* of the resultant; and when they are mutually *at right angles* they are each called the *resolved part* of the resultant in their own direction.

94. In fact it will be found that all definitions and propositions applicable to distances may be applied to velocities (and also to any *vector* quantity). Thus we have

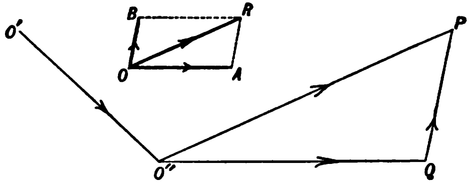
95. PROP. **The parallelogram of Velocities.** *When two velocities are represented (in direction and magnitude) by two straight lines* OA *and* OB, *their resultant is represented (in direction and magnitude) by* OR *the diagonal of the parallelogram* AOB.

Let a point P have (in addition to any other motion) the two simultaneous velocities represented by OA and OB.



Let OA represent u' velos; let OB represent u'' velos; and let OR represent u velos.

Let O' represent the position of the moving point at the beginning of an interval t seconds.



Draw $O'O''$ to represent the distance due to all the other motions of the moving point for the interval t seconds.

Draw $O''Q$ parallel to OA to represent $u't$ ft. and draw QP parallel to OB to represent $u''t$ ft.

Then P is the position of the moving point at the end of t seconds; $O''Q$ is the distance due to the velocity OA; QP the distance due to the velocity OB; therefore $O''P$ is the distance due to their resultant velocity.

But $$\frac{O''Q}{QP} = \frac{u'}{u''} = \frac{OA}{OB} = \frac{OA}{OR}.$$

Also, $O''Q$, QP are parallel to OA, AR respectively.

Therefore the triangle $O''QP$ is similar to OAR.

Therefore $O''P$ is parallel and proportional to OR.

Hence $O''P$ represents ut ft.; that is, $O''P$ is the distance due to the velocity OR. In other words OR represents the resultant of the velocities OA and OB. Q. E. D.

96. When the moving point has the two velocities OA and OB only, then in Art. 95, $O'O''$ disappears, and we see that since $O''P$ is always parallel to OR (which is fixed in direction and magnitude) and always represents ut ft., the actual motion of the moving point is the velocity OR.

97. The actual motion of a point which is moving with any number of simultaneous uniform velocities is uniform velocity.

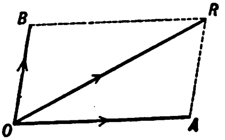
For its resultant velocity, given by the parallelogram of velocities, is always the same; and its resultant distance from its initial position is always parallel and proportional to this resultant velocity. Its actual path *is therefore a straight* line along which it moves with uniform velocity.

98. When a point is moving with given variable velocities, we can find its *actual velocity at any instant* by finding the resultant of the velocities which it has at that instant.

For, if we consider another point Q moving with uniform velocities such that at that instant it has the *same* simultaneous velocities as the point P, then Q will, at that instant, be moving in the same direction with the same speed as the first point P.

99. The Parallelogram of Velocities is sometimes stated in the following form, which is called

The Triangle of Velocities. When a point has two simultaneous velocities, the resultant velocity may be found as follows: Draw a line OA to represent one of the velocities; from A draw a line AR to represent the other velocity; the third side OR of the **triangle** OAR represents the resultant velocity of the two simultaneous velocities OA, AC.



This proposition is only another way of stating the parallelogram of velocities.

For in the parallelogram $OARB$ it is clear that if OA, OB represent two simultaneous velocities, then AR represents a velocity equal and parallel to that represented by OB.

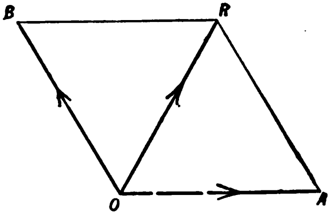
100. To find the resultant of any number of velocities represented by straight lines OA, OB, OC..., we first find OR' the resultant of OA and OB; then we find OR'' the resultant of OR' and OC; and so on.

Or we may proceed thus. Draw from A a line AR' parallel and equal to OB; from R' a line $R'R''$ parallel and equal to OC; and so on: the line joining O to the last point R represents the resultant of all the velocities.

101. When a point at one instant has a certain velocity and at another instant has a different velocity, then some velocity must have been added to its original velocity in the interval.

The velocity which has been added may be found by the parallelogram of velocities; for the new velocity is the resultant of the original velocity and of the velocity added.

Example i. *A point in a certain interval has its velocity changed from* 20 velos *in a given direction to* 20 velos *in a direction making an* angle 60° *with the first direction; find the* additional velocity.



Let OA represent the original velocity; let OR represent the new velocity; join RA; draw OB parallel and equal to AR.

Then OB represents the additional velocity required.

The angle $AOR = 60^\circ$ and $OA = OR$, therefore AOR is an equilateral triangle; hence the angle $ROB = ORA = 60^\circ$.

Therefore $OB = OA$.

Thus, the additional velocity is a velocity of 20 velos in a direction making an angle 120° with the original velocity.

Example ii. *A stone is thrown with* 33 velos *from a train, at right angles to its direction of motion, while the train is going* 30 miles an hour; *find the* initial velocity *of the stone relatively to the ground.*

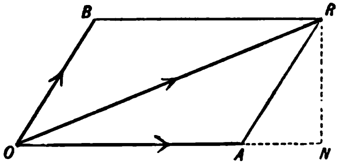
Relatively to the ground the stone has two simultaneous velocities of 33 velos and 44 velos respectively, at right angles to each other.

Therefore, by the parallelogram of velocities, their resultant is a velocity of 55 velos making with the direction of the train an angle whose tangent is $\frac{3}{4}$.

Example iii. *A carriage is going at the rate of* 15 miles an hour; *a man jumps off, giving himself an additional horizontal velocity of* 10 miles an hour; *find his* velocity relative to the ground *when he jumps in a direction making* (i) an angle of 60°, (ii) an angle of 150°, *with the direction in which the carriage is going.*

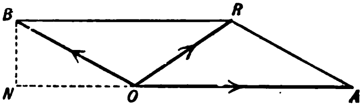
Draw the parallelogram of velocities $AOBR$; draw RN perpendicular to OA; then OR is the required velocity relative to the ground.

Then $$OA=15,\ OB=AR=10.$$



In case (i) $$NAR=AOB=60^\circ;$$
$$AN=AR\cos 60^\circ=\tfrac{1}{2}\times 10;$$
$$NR=AR\sin 60^\circ=\tfrac{1}{2}\sqrt{3}\times 10;$$
therefore, $$OR^2=(OA+AN)^2+NR^2$$
$$=(15+5)^2+(\sqrt{3}\times 5)^2$$
$$=475=(21{\cdot}8\ldots)^2.$$

Therefore the velocity required is 21·8... miles per hour.



In case (ii) $$NAR=30^\circ;\ NA=AR\cos 30^\circ=\tfrac{1}{2}\sqrt{3}\times 10;$$
$$NR=AR\sin 30^\circ=\tfrac{1}{2}\times 10;$$
therefore, $$OR^2=(OA-NA)^2+NR^2$$
$$=(15-5\sqrt{3})^2+5^2$$
$$=(15-5\times 1{\cdot}7320\ldots)^2+25$$
$$=(8{\cdot}18\ldots)^2.$$

Therefore the velocity required is 8·18...miles per hour.

NOTE. When the resultant velocity of a point is zero, the point is at rest.

For the *distance* due to all its velocities after any interval is also zero.

EXAMPLES. XXVII.

Find the resultants of the following 12 pairs of velocities:

1. 12 velos and 16 velos mutually at right angles.

2. 20 velos and 15 velos mutually at right angles.

3. 30 miles an hour and 40 miles an hour at right angles to each other.

4. 2 velos and 8 velos mutually at right angles.

5. a velos and b velos at right angles to each other.

6. a ft. per second and m miles per hour mutually at right angles.

7. 3 velos and 3 velos inclined at an angle of 120°.

8. a velos and a velos inclined at an angle of 60°.

9. 3 velos and $3\sqrt{2}$ velos inclined at an angle of 45°.

10. 10 velos and 20 velos inclined at an angle of 60°.

11. 40 velos and $20\sqrt{3}$ velos inclined at an angle of 150°.

12. 20 velos and 25 velos inclined at an angle whose sine is $\frac{3}{5}$.

13. Find the distance in 10 seconds due to 3 simultaneous velocities of 2 velos, 3 velos and 1 velo making the two angles of 30° and 15° with each other.

14. A particle is given a velocity of 50 velos in a direction making an angle 45° with the horizon; resolve this velocity into two; one horizontal and the other vertical.

15. A particle has velocity 100 velos making an angle whose sine is $\frac{1}{13}$ with the horizon; what is the resolved part of the velocity in a horizontal direction?

16. From a ship sailing at the rate of 25 miles an hour a bullet is fired with a muzzle velocity of 1100 feet per second in a direction at an angle whose sine is $\frac{1}{13}$ with the direction of the ship's motion; find the initial velocity of the shot relative to the water.

17. A football moving with 20 velos receives a kick, after which its direction is changed through a right angle but its speed is unaltered; what velocity was given it by the kick?

18. A particle travelling northwards with 20 velos, receives an additional velocity of 20 velos so that the resultant velocity is 20 velos; in what direction was the additional velocity?

19. The direction of a point's motion is altered by 45° while its speed is unchanged; what is the alteration in its velocity?

20. A cricket ball having 30 velos is struck by a bat so that its velocity is changed to 40 velos in a direction making an angle 150° with its former direction; what velocity was given to the ball by the bat?

21. A stone thrown from a train has a certain velocity given it at right angles to the path of the train, so that relatively to the ground it has 60 velos in a direction making 60^0 with the path of the train; find the velocity of the train.

22. A particle is projected with a velocity whose vertical and horizontal components are 50 velos and 10 velos respectively; the vertical velocity is diminished at the rate of 32 velos per second, the horizontal velocity is unaltered; find (the magnitude and direction of) the velocity of the particle after 1 second. What was the direction in which it was projected?

23. A spherical shot is rolling directly across the smooth horizontal deck of a ship with a velocity 10 velos; find where it would strike the side of the ship supposing the ship, which is going 10 miles an hour, to be suddenly stopped when the shot is 20 ft. from the side.

24. A balloon rising vertically with a velocity of 10 miles an hour is carried by the wind over a horizontal distance of 100 yds. in 20 seconds; find the velocity of the balloon.

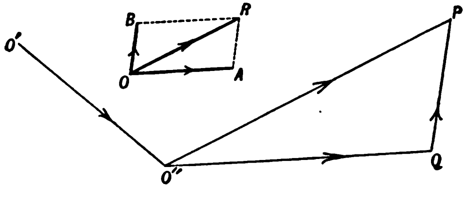
25. A man tricycling westwards at the rate of 8 miles an hour, feels the wind to be due south of him; on increasing his speed to 16 miles an hour the wind appears to be south-west; find the velocity of the wind.

THE PARALLELOGRAM OF ACCELERATIONS.

102. DEF. The **resultant** of two simultaneous *accelerations* is an acceleration, such that the distance due to it after any interval is the resultant of the distances due to the two accelerations.

103. **The parallelogram of accelerations.** *When two simultaneous accelerations are represented (in direction and magnitude) by two straight lines* OA, OB, *their resultant is represented (in direction and magnitude) by* OR, *the diagonal of the parallelogram* AOB.

Let a point have the two simultaneous accelerations represented by OA and OB, and other motions besides.



Let OA represent a' celos;
let OB represent a'' celos;
let OR represent a celos.

Let O' represent the initial position of the moving point.

Let $O'O''$ represent the distance due to all the other motions of the moving point for the interval t seconds.

Draw $O''Q$ parallel to OA to represent $\frac{1}{2}a't^2$ ft.,
draw QP parallel to OB to represent $\frac{1}{2}a''t^2$ ft.

Then P is the position of the point after t seconds; $O''Q$ is the distance due to the acceleration OA; QP is the distance due to the acceleration OB; and therefore $O''P$ is the distance due to their resultant acceleration.

Now $$\frac{O''Q}{QP} = \frac{a'}{a''} = \frac{OA}{OB} = \frac{OA}{AR},$$

also $O''Q$, QP are parallel to OA, AR respectively.

Therefore the triangle $O''QP$ is similar to OAR.

Therefore $O''P$ is parallel and proportional to OR.

Hence, $O''P$ represents $\frac{1}{2}at^2$ ft. and is parallel to OR.

That is, the distance due to the accelerations OA and OB together, viz. $O''P$, is the distance due to the acceleration OR.

Therefore, the acceleration represented by OR is the resultant of the accelerations represented by OA, OB.

104. Art. 103, proves that if we have two points having exactly the same motion except that one of them has the two simultaneous accelerations represented by OA, OB and the other the acceleration represented by OR, their motions will be exactly the same. It proves in fact that the resultant of two accelerations *is* an acceleration.

105. When a moving point is moving with given variable simultaneous accelerations we can find its actual acceleration at any instant by the parallelogram of acceleration.

For, if we consider another point Q moving with uniform accelerations, such that at that instant it has the *same* velocities and accelerations as the first point P, then Q will, at that instant, be moving with the same actual velocity and with the same actual acceleration as the first point P.

106. The resultant of any number of simultaneous accelerations may be found in exactly the same way as the resultant of any number of simultaneous distances or velocities.

107. When the resultant of all the accelerations which a point has is zero, the point is moving with uniform velocity.

For the motion is that of a point having no acceleration.

108. When a point has simultaneous acceleration, each acceleration adds its own velocity to the initial velocity of the point independently of any other velocities which the point may have.

109. The parallelogram of accelerations may be deduced from the parallelogram of velocities in precisely the same words as the parallelogram of velocities was deduced from that of distances by reading velocity for distance and acceleration for velocity in Art. 95.

The parallelogram of acceleration may also be stated as the **Triangle of Accelerations** in the words of Art. 99.

EXAMPLES. XXVIII.

1. A man is starting to leave a railway carriage walking directly towards the door with an acceleration 3 celos, and the train itself is stopping with retardation 4 celos; what is the direction of the resultant acceleration of the man?

2. A man in a train which is getting up its speed with an acceleration 5 celos, gives a ball an acceleration 12 celos, in the direction at right angles to the motion of the train; find the resultant acceleration of the ball.

3. Supposing in Question **2.** the man commences to throw the ball just as the train is starting and he keeps up the pressure on the ball with his hand (thus producing 12 celos) for 1 second, find the initial velocity of the ball on leaving his hand.

4. Supposing in Question **2.** the train is moving at the rate of 30 miles an hour, (and with 5 celos) when the man commences his throw, and that he keeps up the pressure for 1 sec., find the initial velocity of the ball.

5. A man runs across a railway carriage which is 7 ft. across with an acceleration of 5 celos; the carriage is moving with a uniform velocity of 30 miles an hour; find the velocity with which he leaves the carriage.

6. A train is pulling up with a retardation of 10 celos; I wish to walk straight across the carriage with an acceleration of 10 celos; in what direction shall I have to apply a horizontal force to my body in order to do so?

CHAPTER VIII.

THE PARALLELOGRAM OF FORCES.

110. DEF. The **resultant** of two *forces* is that *force* which acting on any mass will produce in that mass the same acceleration as the two forces would produce when acting together on the same mass.

111. The two forces which together are equivalent to a resultant are called its **components.** When components are mutually *at right angles* they are each called *the resolved parts* of their resultant in their own direction.

112. PROP. **The parallelogram of Forces.** *When two forces acting on a particle are represented* (*in direction and magnitude*) *by the straight lines* OA, OB, *their resultant is represented* (*in direction and magnitude*) *by* OR *the diagonal of the parallelogram* AOB.

For, by Art. 33, when two forces act on a particle each force produces in the particle its own acceleration in its own direction; and since the forces are acting on the same mass, the accelerations produced are proportional to the forces producing them.

The resultant of the two forces must therefore be such that it produces in the same particle the resultant of these two accelerations.

Therefore the resultant force is parallel and proportional to the resultant acceleration.

That is, the two forces and their resultant are proportional to and in the same direction as the two accelerations and their resultant.

Therefore, if we represent the two forces by the two straight lines OA, OB, then the diameter OR of the parallelogram AOB represents their resultant. Q. E. D.

113. In considering the motion of a mass acted on by several forces either

I. We may consider that each force produces its own acceleration in the mass, and then find the acceleration of the mass by finding the resultant of all these accelerations, or

II. We may first find the resultant of all the forces, and then consider that the forces are equivalent to this resultant only, which resultant produces the actual acceleration of the mass.

The second method is the one usually adopted.

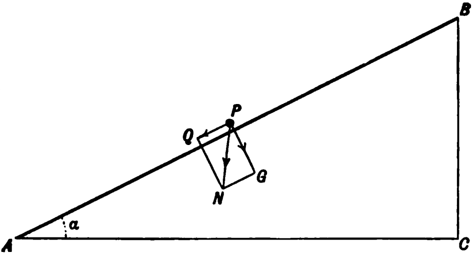
114. It should be noticed that the three propositions called the Parallelogram of Distances, the Parallelogram of Velocities, and the Parallelogram of Acceleration are *geometrical facts*; while the proposition known as the Parallelogram of Forces is deduced from the Parallelogram of Accelerations, by means of the definition of Force as that which produces acceleration in mass, each force producing its own acceleration exactly as if the others did not exist.

115. When the resultant of any number of forces is zero, they produce no resultant acceleration; in other words, they have no effect on the motion of any mass on which they simultaneously act.

Forces whose resultant is zero are said to be in **equilibrium**.

116. As a particular case we may notice, that when one of three forces is equal and opposite to the resultant of the other two, the three forces are in equilibrium.

Example i. *A mass* m pounds is *placed under the action of gravity on a smooth inclined plane; find its acceleration.*



Let AB be the inclined plane; draw AC horizontal and BC vertical.

Let the angle BAC be α.

The weight of the mass P is a force mg poundals vertically downwards; let PN represent this force.

Resolve this force into its two components PQ and PG, in the directions along and perpendicular to PA respectively.

Then instead of mg poundals vertically downwards we have

$$PQ = mg \sin \alpha \text{ poundals along } PA$$

and

$$PG = mg \cos \alpha \text{ poundals along } PG.$$

Now the presence of the plane prevents any motion in the mass in the direction PG.

Also, the plane is '**smooth**'; by which is meant, that the stress between the mass and the plane must be perpendicular to the plane.

These conditions will both be satisfied if we suppose that the plane exerts a pressure on the mass, in the direction GP, just sufficient to prevent the mass from having any motion in that direction.

Hence we say, that the pressure of the plane on the mass in the direction GP is $mg \cos \alpha$ poundals.

Thus, the forces acting on the mass are $mg \sin \alpha$ poundals along PA, $mg \cos \alpha$ poundals along PG and $-mg \cos \alpha$ poundals in the same direction.

Therefore, the resultant force is $mg \sin \alpha$ poundals along PA.

This force produces in m pounds the acceleration in the direction PA

$g \sin \alpha$ *celos.*

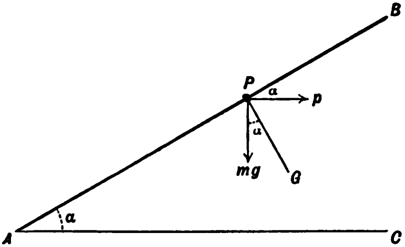
Example ii. *A mass* m lbs. *is placed on a smooth inclined plane and is acted on by its own weight and by a horizontal force* p poundals; *find* its acceleration.

With the construction of Example i. resolve the weight and the force p poundals along and perpendicular to the plane.

Case (i) We have if p poundals act as the forces in figure I.

$$(mg \sin \alpha - p \cos \alpha) \text{ poundals, along } PA,$$

$$(mg \cos \alpha + p \sin \alpha) \text{ poundals} - \text{the pressure of the plane, along } PG.$$



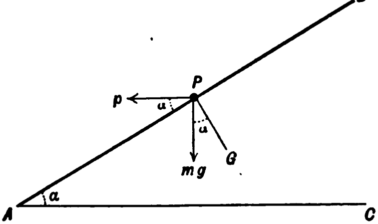
Assuming that the resultant acceleration is along AB, it follows that the resultant force must be along AB; so that

the pressure of the plane on the mass $= (mg \cos \alpha + p \sin \alpha)$ poundals; therefore the resultant force is $(mg \sin \alpha - p \cos \alpha)$ poundals along PA, which produces in mass m lbs. the acceleration

$$\left(g \sin \alpha - \frac{p \cos \alpha}{m}\right) \text{ celos along } PA.$$

If this acceleration happens to be zero the forces are in equilibrium; that is, if $mg \sin \alpha = p \cos \alpha$.

Case (ii) if p act as in figure II., then as before,



the pressure of the plane on the mass $=(mg\cos\alpha - p\sin\alpha)$ poundals.

And the acceleration along PA

$$\left(g\sin\alpha + \frac{p\cos\alpha}{m}\right) \text{ celos.}$$

EXAMPLES. XXIX.

1. Forces of 3 poundals and 4 poundals at right angles to each other act upon a mass of 5 lbs.; find the acceleration produced. How long will the mass take to move 18 ft. from rest?

2. Equal forces of 10 poundals at an angle of 120^0 to each other act on a mass of 1 lb.; how long will the mass take to acquire a velocity of 30 miles an hour?

3. A mass of m lbs. is let fall from rest under the action of gravity and is acted on by a horizontal force of 24 poundals; find how long it will take to fall a vertical distance of 144 ft.; and how far it will move horizontally in the same time.

4. Two horses, attached by ropes to a mass placed on a smooth horizontal plane and weighing 1 ton, pull at right angles to each other, one with a force equal to the weight of 3 cwt. and the other with a force equal to the weight of 4 cwt.; find the direction of motion and the acceleration of the mass.

5. If in Question **4** the plane is rough so that there is a force of friction which retards the motion equal to $\frac{1}{10}$th of the pressure of the mass on the plane; find the acceleration.

6. A particle slides down a smooth inclined plane whose inclination to the horizon is 30^0; shew that it takes twice as long to descend a given vertical distance as a particle falling freely under the action of gravity.

7. Forces of 2 poundals, 3 poundals and 1 poundal act along lines OA, OB, and OC so that the angle $AOB = 45^\circ = BOC$ upon a mass of 4 lbs.; find the acceleration produced.

8. Forces of 2 poundals, 3 poundals and 1 poundal act, along lines OA, OB and OC, so that the angle $AOB = 30^0$, $BOC = 15^0$, upon a mass of 1 lb.; find the acceleration produced.

9. Prove that the mass in Example **3** moves in a straight line.

10. A mass P is drawn up a smooth plane inclined at an angle of 30^0 to the horizon by means of an equal mass Q which descends vertically, the masses being connected by a string which passes over a small pulley at the top of the plane; prove that the acceleration is $\frac{1}{4}g$ celos.

11. Find the ratio of the masses of P and Q in Question **10** that they may have no acceleration.

12. Suppose that P in Question **10** starts from rest at the bottom of the plane and the interval t seconds occupied by Q in drawing P to the top of the plane is noticed; then P and Q being interchanged the interval t' seconds occupied by P in drawing Q up the plane is noticed; shew that if $t=2t'$ then $Q=\frac{3}{2}P$.

13. A mass m lbs. is drawn up a smooth plane inclined at an angle α to the horizon, by means of a mass m' lbs. which descends vertically, the masses being connected by a string which passes over a small pulley at the top of the plane; prove that the acceleration is

$$\frac{m\sin\alpha - m'}{m+m'}\,g \text{ celos.}$$

14. Two masses m lbs. and m' lbs. are connected by a light string which passes over a small smooth pulley fixed at the top of two smooth inclined planes of equal heights placed back to back, the inclination of the planes being each 30^0; shew that when the masses are equal they have no acceleration.

15. In Question **14** shew that when $m'=\frac{1}{2}m$ the acceleration is $\frac{1}{6}g$ celos.

16. If in Question **14** the inclinations of the planes to the horizon are α and α' respectively, prove that the acceleration is

$$\frac{m\sin\alpha - m'\sin\alpha'}{m+m'}\,g \text{ celos,}$$

and that the tension of the string is

$$\frac{mm'(\sin\alpha+\sin\alpha')}{m+m'}\,g \text{ poundals.}$$

17. Determine the acceleration due to gravity from the following experiment. A mass descending vertically under the action of gravity draws an equal mass a distance of 25 ft. from rest in $2\frac{1}{2}$ seconds up a smooth plane inclined at an angle 30^0 to the horizon, by means of a light string passing over a smooth small pulley at the top of the plane.

Example iii. *To find the* interval occupied *by a particle in sliding down a smooth inclined plane of length* s ft. *and inclination* α.

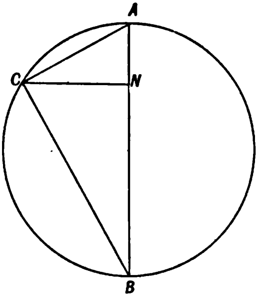
By Example i. the particle has an acceleration $g \sin \alpha$ celos down the plane.

The interval t seconds, occupied by a point in moving from rest over a distance of s ft. with acceleration $g \sin \alpha$ celos, is given by

$$s = \tfrac{1}{2}(g \sin \alpha)t^2,$$

whence,
$$t = \sqrt{\left\{\frac{2s}{g \sin \alpha}\right\}}\,.$$

Example iv. *A series of* chords *are drawn from the highest point of a fixed vertical circle; prove that the intervals occupied by a particle in sliding from rest down each of these chords* (supposed smooth) *are all equal.*



Using the notation of Example iii, let ABC be the circle; AC one of the chords; CN horizontal; then $AC = s$ ft. and $ACN = \alpha$.

The interval occupied in sliding down AC, by Example iii, is

$$\sqrt{\left\{\frac{2s}{g \sin \alpha}\right\}} \text{ seconds.}$$

But
$$AC = AB \sin \alpha,$$

That is, $\dfrac{s}{\sin \alpha} = AB = 2r$; the length of the radius being r feet.

Hence, the interval in question is

$$2\sqrt{\frac{r}{g}} \text{ seconds,}$$

which is the same for each chord. Q. E. D.

EXAMPLES. XXX.

1. Prove that the interval occupied by a particle in sliding down an inclined plane of height h ft. and inclination α is

$$\frac{1}{\sin\alpha}\sqrt{\left\{\frac{2h}{g}\right\}}.$$

2. A train is on a smooth incline of 1 in 100; find how long it will take to go a mile from rest. [N.B. by an incline of 1 in 100 is meant that in the figure of page 91, the incline is such that $BC=1$ and $AB=100$.]

3. Find the velocity acquired at the end of the mile by the train in Example **2**. If another train start to go up a smooth incline of 1 in 50 with that velocity, shew that it will go half a mile up the plane.

4. A particle slides down a smooth straight tube of length l inclined at an angle α to the horizon, and then falls freely under the action of gravity; if the lower end of the tube is h feet above a horizontal plane, find when and where the particle will strike the plane.

5. A particle slides down a smooth straight tube, whose upper point is at a height h ft. from a horizontal plane, and then falls freely under the action of gravity; prove that it will in all cases reach the horizontal plane with $\sqrt{(2gh)}$ velos.

6. A particle is sliding down a smooth tube inclined at an angle to the horizon, and it takes twice as long to descend any vertical distance as a particle falling freely; find the inclination of the tube.

7. Find the direction in which a smooth tube must be drawn, so that one end being at a fixed point and the other end on a fixed straight line, a particle sliding down it may reach the straight line in the shortest possible time.

NOTE. The above problem is often stated thus. Find the line of **quickest descent** from a given point to a given straight line.

8. Prove that the time of sliding down any chord, supposed smooth, of a vertical circle drawn to the lowest point, is constant.

9. Find the line of quickest descent from a given straight line and a given point.

*SECTION III.

ILLUSTRATIONS.

CHAPTER IX.

PROJECTILES.

117. A particle which, having been projected from any point in any direction, is then supposed to move under the action of its own weight only, is called a **projectile.**

The resistance of the air is here neglected. This however has a very considerable effect on the motion of any mass actually projected, and renders this solution of the problem of little practical use.

118. When a projectile is projected vertically upwards its path is a vertical straight line.

This case has already been considered [Art. 46].

119. When a particle is projected in a direction which is not vertical, its path is a curved line.

This curved line is in one plane, which contains the vertical and the original direction of projection.

For there is no force tending to take it out of this plane.

120. We shall denote the velocity of projection by u velos, and shall suppose that its direction makes an angle α (radians) with the horizon.

121. The resolved parts of the velocity of projection are

(i) $u \cos \alpha$ (velos) in the horizontal direction,

(ii) $u \sin \alpha$ (velos) in the vertical projection.

122. PROP. *The* horizontal velocity *of a projectile is* constant.

By the *horizontal velocity* is meant *the resolved part in a horizontal direction* of the velocity at any instant.

The acceleration of the particle is always *vertically* downwards; so that after any interval, the *additional* velocity of the particle is also vertically downwards, and consequently has no resolved part in the horizontal direction.

The horizontal velocity is therefore unaltered. Q. E. D.

123. PROP. *The* vertical velocity *of a projectile after* t seconds *is* $(u \sin \alpha - gt)$ velos.

This is the expression of the fact that initially the particle has a vertical velocity $u \sin \alpha$ velos, and that it has the vertical acceleration $-g$ celos.

124. PROP. *The* velocity *of a projectile after* t seconds *is* $\sqrt{\{(u \cos \alpha)^2 + (u \sin \alpha - gt)^2\}}$ velos,

and if θ *be the angle which its direction makes with the horizon*

$$\tan \theta = \frac{u \sin \alpha - gt}{u \cos \alpha}.$$

For, the velocity of the projectile at any instant is the resultant of the horizontal and vertical velocities at that instant [See Art. 95].

125. PROP. *To find when and where a projectile is at its greatest vertical height.*

The vertical velocity is $(u \sin \alpha - gt)$ velos.

When this is positive the particle is going upwards, and when this is negative it is going downwards, and when this is zero it has reached its greatest height; that is, when

$$t = \frac{u \sin \alpha}{g}.$$

The vertical distance is

$$(u \sin \alpha t - \tfrac{1}{2} g t^2) \text{ feet,}$$

in this putting $\dfrac{u \sin \alpha}{g}$ for t, we find for the greatest vertical distance $\dfrac{u^2 \sin^2 \alpha}{2g}$ feet.

The horizontal distance at the same instant is found by putting $\dfrac{u \sin \alpha}{g}$ fo[illegible] in $u \cos \alpha t$; it is

$$\frac{u^2 \cos \alpha \sin \alpha}{g} \text{ feet.}$$

EXAMPLES. XXXI.

1. A particle is projected under the action of gravity with 25 velos making an angle whose tangent is $\frac{3}{4}$ with the horizon; find when it is moving with 20 velos.

2. A particle is projected with 416 velos making an angle with the horizon whose sine is $\frac{12}{13}$; shew that its least velocity is 160 velos, and that when it has that velocity it is moving horizontally.

3. Find when the projectile in Example **2** has a velocity of 384 velos; find also its direction of motion at that instant.

4. Shew that the total velocity of a projectile projected with given initial velocity at a given angle is least when it is moving horizontally.

5. A projectile is projected with u velos making an angle α with the horizon; prove that it will be moving in a direction at right angles to the direction of projection after $\frac{u}{g}$ cosec α seconds.

6. Two particles are projected simultaneously in the same vertical plane with velocities u velos and v velos at angles α and β to the horizon respectively; shew that their velocities will be parallel after

$$\frac{uv \sin (\alpha - \beta)}{g (v \cos \beta - u \cos \alpha)} \text{ seconds.}$$

7. A particle projected with u velos at an angle α, is moving in a direction making an angle β with the horizontal plane after

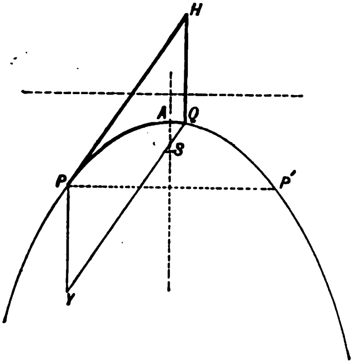
$$\frac{1}{g}(u \sin \alpha - u \cos \alpha \tan \beta) \text{ seconds.}$$

126. PROP. *The* path *of a projectile is a* **Parabola.**

Let u velos be the initial velocity making an angle α with the horizon.

Let P be the point of projection; Q the position of the projectile after t seconds. Draw PP' horizontal; draw PV vertically downwards; draw PH making an angle α with PP'; draw QH vertical and QV parallel to HP.

Then [Art. 91] $PH = ut$ ft.; $HQ = \frac{1}{2}gt^2$ ft.



$$\text{Therefore} \qquad QV^2 = PH^2 = u^2t^2$$

$$= u^2 \times \frac{2HQ}{g} = \frac{2u^2}{g} \times PV$$

$$= 4SP \, . \, PV; \qquad \text{where } 4SP = \frac{2u^2}{g} \text{ ft.}$$

But in the Parabola, $QV^2 = 4SP \, . \, PV$; where SP is the distance of the focus from P.

Therefore, if we draw a parabola, touching PH at P, whose axis is vertical and vertex upwards, such that

$$4SP = \frac{2u^2}{g} \text{ ft.,}$$

then Q will always be on this parabola. Q. E. D.

127. PROP. *The* speed *of a projectile at any point of its path is equal to that due to a fall from the directrix.*

Let S be the focus and P the point of projection, then, SP is the vertical distance of P from the directrix; and by the last Article, $SP = \frac{u^2}{2g}$ ft.

Let h be the number of ft. in SP; then, the velocity v due to falling with acceleration g, a distance h ft. is given by

$$v^2 = 2gh$$

but from above $$u^2 = 2gh.$$

Thus the speed of projection is equal to that due to falling from the directrix.

And if this is true at the point of projection it must be true at every point; for the particle might be projected from any point in its path.

NOTE. This proposition may be stated as follows; When any particle has fallen vertically under the action of gravity from a point on the directrix to a point on the parabola it has a vertical velocity downwards, whose *magnitude* is equal to the velocity which the particle describing the parabola has when at that point. It must be noticed that these two velocities are not in the same direction.

It follows that the horizontal speed is that due to falling from the directrix to the vertex, and hence that the number of feet in the latus-rectum of the parabola is $\frac{2u^2 \cos^2 \alpha}{g}$.

128. Another proof of Art. 127 is as follows.

Let the tangent at P cut the axis of the parabola in T. The vertical velocity at A is zero; so that the vertical velocity at $P\,(u \sin \alpha)$ is that due to falling a vertical distance AN; hence $u^2 \sin^2 \alpha = 2g\,AN$.

[Now in the parabola with the usual notation

$$2AN = NT = PT \sin \alpha = 2PY \sin \alpha = 2SP \sin^2 \alpha],$$

whence $$u^2 \sin^2 \alpha = 2g\,SP \sin^2 \alpha,$$

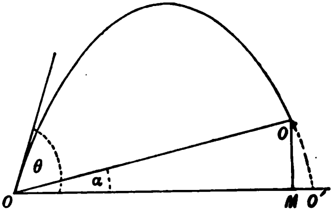
or $$u^2 = 2g\,SP.$$ Q. E. D.

Example i. *To find the* range *and* time of flight *on an inclined plane,* angle α, *which passes through the point of projection.*

Resolve the velocity of projection along and perpendicular to the inclined plane; the resolved parts are, in velos,

initially, $u\cos(\theta-\alpha)$; $u\sin(\theta-\alpha)$;

after t secs, $u\cos(\theta-\alpha)-g\sin\alpha\times t$; $u\sin(\theta-\alpha)-g\cos\alpha\times t$.



The distances resolved in the same directions after t secs., are in ft.,

$$u\cos(\theta-\alpha)t-\tfrac{1}{2}g\sin\alpha t^2;\ u\sin(\theta-\alpha)t-\tfrac{1}{2}g\cos\alpha t^2.$$

The particle strikes the plane again when the distance perpendicular to the plane viz. $u\sin(\theta-\alpha)t-\frac{1}{2}g\cos\alpha t^2$ is zero;

that is, when $$t=\frac{2u\sin(\theta-\alpha)}{g\cos\alpha}.$$

Let OO'' in the figure be the range; draw $O''M$ perpendicular to the horizontal line OO'; then OM = horizontal velocity × time of flight

$$=u\cos\theta\times\frac{2u\sin(\theta-\alpha)}{g\cos\alpha},$$

also, $$OO''=\frac{OM}{\cos\alpha},$$

that is, the range is $$\frac{2u^2}{g}\frac{\sin(\theta-\alpha)\cos\theta}{\cos^2\alpha}\text{ feet.}$$

Example ii. *Prove that in Example* i. *the particle at the middle of the time of flight is vertically above the middle point of the range and is moving parallel to the inclined plane.*

The first statement is most easily proved by observing that if the projectile carries a fine thread which hangs always vertical, this thread travels with uniform velocity in a horizontal direction; and that therefore the point in which this vertical thread cuts the inclined plane travels with *uniform velocity along the plane.*

To prove the second statement, we observe that the velocity of the projectile *perpendicular* to the inclined plane is zero, when

$$u \sin(\alpha - \beta) - g \cos\beta \times t \text{ is zero,}$$

that is, when $$t = \frac{u \sin(\alpha - \beta)}{g \cos\beta}.$$

which is at the middle of the time of flight.

Therefore at the middle instant of the time of flight the velocity of the projectile is parallel to the plane.

EXAMPLES. XXXII.

1. A train is moving at the rate of 45 miles an hour, when a ball is dropped from the roof inside one of the carriages; prove that the ball describes a parabola in space, and find the position of the axis and directrix.

2. A boy throws a stone from the top of a cliff 128 ft. high with 64 velos at an angle 30° to the horizon; find how far from the foot of the cliff it will strike the sea and with what velocity.

3. Prove that, neglecting the resistance of the air, the range of a projectile on a horizontal plane is greatest when the angle of projection is 45°.

4. Find with what velocity a man must be able to throw that he may just be able to throw a cricket ball 100 yards.

5. A man throws a ball with a velocity 50 velos; in what direction must he throw it that he may just strike the top of a pole 40 ft. high at a distance of 100 ft. from him?

6. A particle is projected with velocity u velos at an elevation α; find its least velocity, and find when and where it attains that velocity.

7. A particle is describing a parabola under the action of gravity, and at the instant when it has its least velocity that velocity is doubled; shew that the time of reaching the horizontal plane from which it was projected is unaltered.

8. From the top of a tower I project particles in a horizontal direction; prove that the distances, from the foot of the tower at which the particles strike the horizontal plane on which it stands, are proportional to the velocities of projection.

9. A projectile has initially u velos and its angle of projection is α; prove that its distance from the point of projection after t seconds is

$$(u^2t^2 - u \sin\alpha\, gt^3 + \tfrac{1}{4} g^2t^4) \text{ feet.}$$

10. Find the angle of elevation that the horizontal range may be equal to the distance of the point of projection from the directrix.

11. Shew that the greatest range on an inclined plane of 30°, is two-thirds of the greatest range on a horizontal plane, with the same initial velocity.

12. A straight smooth tube *AB*, 72 ft. long, is placed at an angle 30° to the horizon so that its lower end *B* is at a height 16 ft. from a horizontal plane; a particle is allowed to slide from *A* through the tube and then to describe a parabola freely; find how long after leaving the tube the particle strikes the ground.

13. In Question **12**, find how far from the point vertically under *B* is the point at which the particle strikes the ground.

14. A straight smooth tube *AB*, a ft. long, inclined at an angle α to the horizon is fixed so that the lower end *B* is h ft. from a horizontal plane. A particle slides from *A* through the tube and then describes a parabola freely, striking the plane at *C*; find the time of falling from *B* to *C* and the distance *BC*.

15. A man in a railway carriage, moving with uniform velocity 30 miles per hour, throws a ball so that it goes up four feet and returns to the point of projection; shew that the ball describes a parabola relatively to the earth and find how far the train moves while it does so.

16. Two men 5 ft. apart in a railway carriage which is moving uniformly with a velocity of 45 miles an hour toss a ball from one to the other, projecting it so that its time of flight is ½ second. Shew that the ball describes a parabola in space; and taking its path in the carriage to be perpendicular to the rails, find its range in space.

17. The top of a railway carriage is 12 ft. from the ground, and it is moving at the rate of 7½ miles per hour; a man jumps from the top, giving himself a horizontal velocity perpendicular to the rails of 8¼ velos; find the velocity with which he will reach the ground.

18. A particle is projected with velocity u at an angle α; with what velocity must I move on a horizontal plane that I may be always vertically under the projectile?

19. A particle is projected from a point on an inclined plane of angle β with velocity u at an angle α to the horizon so that its path is in the same plane as the line of greatest slope in the plane; with what velocity must I move on the plane so as to be always vertically under the particle?

20. **Morin's machine** consists of a vertical circular cylinder which is made to rotate with uniform velocity, while one of the weights of an Atwood's machine having a pencil fixed to it descends with constant acceleration in a vertical line, and as it does so marks a line on the surface of the cylinder; shew that if the cylinder is covered with a sheet of paper, which after the pencil has made its mark upon it, is taken off and flattened, then the trace of the pencil on the paper will be a parabola.

21. Find the length of the latus rectum of the parabola described on a Morin's machine, in which the circumference of the cylinder is 3 ft. and the weights in the Atwood's machine are $15\frac{1}{2}$ oz. and $16\frac{1}{2}$ oz. respectively [$g=32$], the cylinder making a complete revolution in 6 seconds.

22. A particle is projected with velocity u velos at an angle α; shew that it reaches the further extremity of the latus rectum in

$$\frac{1}{g}(u \sin \alpha + u \cos \alpha) \text{ seconds.}$$

23. A particle slides down a smooth inclined plane of angle α, having initially a horizontal velocity on the plane (in a direction perpendicular to the line of greatest slope) of u velos; shew that the particle traces out a parabola on the plane and find its latus rectum.

24. A particle is projected with velocity u velos at an angle α; shew that the focus is $\frac{u^2}{2g}(\sin^2\alpha - \cos^2\alpha)$ above the horizontal plane through the point of projection.

25. The focus of the path of a projectile is above or below the horizontal plane through the point of projection according as the vertical component of the velocity of projection is greater or less than the horizontal component.

26. A particle slides down a smooth straight tube and then falls freely under the action of gravity; prove that the directrix of its parabolic path passes through the upper end of the tube.

CHAPTER X.

OBLIQUE IMPACT.

129. An impact which is not *direct*, as defined in Art. 65, is said to be **oblique.**

130. When two particles impinge obliquely, the line of action of the stress between the particles set up by the impact is called the **line of impact.**

131. In order to find the velocity of the particles after impact, the velocity before impact of each particle must be resolved into two *resolved parts*, one along, the other perpendicular to, the line of impact;

Then, since the stress is in the line of impact, it has no effect on either of the resolved parts of the velocities perpendicular to this line.

The stress produces its effect on the resolved parts of the velocities in the line of impact exactly as if the other resolved parts did not exist.

So that we find the velocities in the line of impact after impact as in Art. 72.

Each of these new velocities is then compounded with the resolved part perpendicular to the line of impact (which is unaltered) of the velocity before impact.

The resultant velocities thus obtained are the required velocities after impact of the particles respectively.

Example. Two spheres A *and* B, *of* m lbs. *and* m′ lbs. *having velocities* u velos *and* u′ velos, *making* angles α *and* α' *respectively with the line of impact, impinge on one another; find their* velocities *after impact.*

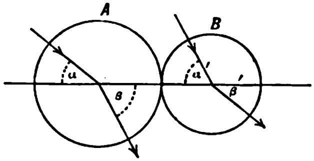
The resolved parts of the velocities perpendicular to the line of impact are in velos, $u \sin \alpha$, $u \sin \alpha'$, respectively.

These are unaltered by the impact.

The resolved parts of the velocities along the line of impact are

$$u \cos \alpha, \quad u' \cos \alpha', \text{ respectively.}$$

Let the elasticity of the spheres be e.



Then, since the problem is now one of direct impact, we have, by Arts. 68 and 72, if v, v' be the velocities after impact in the line of impact

$$mv + m'v' = mu \cos \alpha + m'u' \cos \alpha',$$

and,

$$v - v' = -e\,(u \cos \alpha - u' \cos \alpha')$$

whence,

$$v = \frac{(m - em')\, u \cos \alpha + m'\,(1 + e)\, u' \cos \alpha'}{m + m'},$$

$$v' = \frac{(m' - em)\, u' \cos \alpha' + m\,(1 + e)\, u \cos \alpha}{m + m'}.$$

The resultant velocity of A is therefore $\sqrt{\{u^2 \sin^2 \alpha + v^2\}}$ in the direction making the angle whose tangent is $\frac{u \sin \alpha}{v}$ with the line of impact; a similar statement gives the velocity of B.

EXAMPLES. XXXIII.

1. A sphere A of elasticity e impinges with $20\sqrt{2}$ velos on an equal sphere B at rest, the line of impact making an angle of 45^0 with the direction of motion of A; find the velocity of B after impact.

2. A sphere impinges on an equal sphere at rest, find the condition that after impact their velocities may be at right angles.

3. A sphere A impinges on a sphere B of equal mass; their velocities before impact are at right angles and equally inclined to the line of impact and are equal in magnitude; shew that when $e = \frac{1}{3}\sqrt{3}$ their velocities after impact are inclined at an angle 60°.

4. A glass ball of elasticity e when moving horizontally with velocity u velos, is struck by an equal ball moving vertically with u' velos, so that the line of impact is vertical; find their velocities after impact.

5. A bird weighing 1 lb. moving horizontally with 80 velos is struck by a bullet weighing 1 oz. moving vertically with a velocity of 1020 velos; find the subsequent velocity of the bird supposing the bullet to lodge in the bird; and supposing it killed by the shot when at a height of 100 ft., find when it will fall to the ground, neglecting the resistance of the air.

6. A bird weighing 2 lbs. moving horizontally with 30 velos is struck by a bullet of 1 oz. moving horizontally, but at right angles to the path of the bird, with a velocity 1320 velos; find the velocity after impact supposing the bullet to lodge in the bird; and supposing the bird killed when at a height of 128 ft., find how long it will be before it strikes the ground.

132. When an elastic ball impinges obliquely on a plane fixed to the ground, the problem is treated as in Art. 73.

Examples. A sphere *of* elasticity e *is projected with* velocity u *at an* angle α to the horizon *from a point in a smooth horizontal plane, against which it impinges and rebounds; investigate the motion.*

The resolved parts of the velocity of projection are (in velos)

$u \sin \alpha$, vertically; $u \cos \alpha$, horizontally.

The particle first describes a parabola.

On reaching the horizontal plane again the particle has velocity

$-u \sin \alpha$, vertically; $u \cos \alpha$ horizontally.

The impact does not alter the horizontal velocity; the vertical velocity after impact is $eu \sin \alpha$.

The particle now describes another parabola the velocity of projection being $u \cos \alpha$ velos horizontally, $eu \sin \alpha$ velos vertically. And so on.

Thus, after the nth impact the vertical velocity is

$e^n u \sin \alpha$; the horizontal velocity is $u \cos \alpha$.

The *range* of any rebound is (in feet)

$$\frac{2\,(\text{vertical velocity} \times \text{horizontal velocity})}{g}.$$

The distance from the point of projection at which the sphere will strike the plane after the nth rebound is

$$\frac{2u^2 \sin a \cos a}{g} \{1 + e + e^2 + \ldots + e^n\};$$

the limit of which, when n is increased without limit, is

$$\frac{2u^2 \sin a \cos a}{g(1-e)}.$$

The meaning of this is, that after reaching this distance from the point of projection, the sphere will cease to rebound and will slide with velocity $u \cos a$.

133. The following proposition is important.

PROP. *When two* masses *of* m lbs. *and* m′ lbs. *respectively of* elasticity e *impinge directly, if* v velos *and* v′ velos *be their velocities after impact then* $mv^2 + m'v'^2$ *is less than* $mu^2 + m'u'^2$.

We have, $$mv + m'v' = mu + m'u'$$

and, $$v - v' = -e(u - u').$$

Therefore $$(mv + m'v')^2 = (mu + m'u')^2$$

and, $$mm'(v - v')^2 = mm'e^2(u - u')^2.$$

Whence, by addition

$$(m + m')(mv^2 + m'v^2) = (m + m')(mu^2 + m'u^2) - mm'(1 - e^2)(u - u')^2,$$

or, $$mv^2 + m'v^2 = mu^2 + m'u^2 - \frac{m\,m'}{m + m'}(1 - e^2)(u - u')^2.$$

Now e is never greater than 1; so that $1 - e^2$ cannot be negative; hence $mv^2 + m'v'^2$ is always less than $mu^2 + m'u^2$; except when $e = 1$, and in that case the two expressions are equal.

This result may be expressed (See Chapter XV.) thus; *The* kinetic energy *of two masses is diminished by direct impact, except when the* elasticity *of the masses is perfect.*

We leave as an exercise for the student the proof of the proposition that if v velos and v' velos be the velocities after oblique impact of masses m lbs. and m' lbs. then $mv^2 + m'v'^2$ is less than $mu^2 + m'u'^2$.

EXAMPLES. XXXIV.

1. A glass ball of elasticity $\frac{2}{3}$rds is projected with 40 velos at an angle of projection whose sine is $\frac{4}{5}$, from a point on a horizontal pavement; find its range after one rebound.

2. A glass ball of elasticity $\frac{1}{2}$ is projected at an angle whose sine is $\frac{12}{13}$ with 156 velos from a point in a horizontal plane; find how far it will go before it ceases to rebound.

3. A particle of elasticity $\frac{3}{5}$ is projected with velocity 100 velos at an angle 60^0 to the horizon from a point in an inclined plane making an angle 30^0 with the horizon; find the velocity of the particle after one rebound from the plane.

4. A particle of elasticity e is projected with u velos at an angle α to the horizon and after striking a fixed vertical wall at a horizontal distance h ft. returns to the point of projection; prove that

$$hg(1+e) = 2u^2 e \sin\alpha \cos\alpha.$$

5. Two particles of elasticity e are projected in exactly opposite directions from a point between two fixed vertical parallel planes and distant a ft. and b ft. from them respectively; shew that they will not meet again unless the directions of projection are perpendicular to the planes.

6. A particle of elasticity e is projected from a point half-way between two fixed parallel vertical walls $2a$ ft. apart, in a given direction, and after three rebounds returns to the point of projection; find the velocity of projection.

NOTE. When a sphere strikes a plane the angle its velocity makes with the normal to the plane is called the **angle of incidence**; the angle which its velocity after impact makes with this normal is called the **angle of reflexion.**

7. A sphere of elasticity $\frac{1}{4}$ impinges on a plane; find the angle of incidence that its direction after impact may be at right angles with its direction before impact.

8. Prove that when a perfectly elastic sphere impinges on a smooth plane the angles of incidence and reflexion are equal.

9. A perfectly elastic ball is projected on a smooth rectangular billiard table in a direction parallel to one of its diagonals; find the condition that after impinging on each of the four sides the ball will return to the point of projection.

10. If α be the angle of incidence and β the angle of reflexion of a sphere of elasticity e on a smooth plane, prove that $\cot\beta = e\cot\alpha$.

11. A sphere sliding on a smooth horizontal plane impinges in succession on two smooth vertical planes at right angles to each other; prove that the velocity of the sphere after the second impact is parallel to its velocity before the first impact.

12. Two spheres moving in parallel directions with equal and opposite mass-velocities impinge; prove that after impact they will move in parallel directions with velocities in the inverse proportion of their masses.

13. A smooth small sphere of elasticity e slides down a smooth inclined plane of height h and inclination α, and impinging on a smooth horizontal plane at the foot of the first describes a parabola; find the range.

14. A particle of elasticity e is projected from a horizontal plane so as to impinge on a given vertical wall at a distance a ft., and after one rebound on the horizontal plane returns to the point of projection; the velocity of projection being u velos, find the direction of projection.

15. A sphere of elasticity e is projected at an angle α from the middle of the floor of a rectangular room h ft. high, $2b$ ft. wide, so that after impinging on a wall, on the ceiling and on the opposite wall, it returns to the point of projection; find equations to determine the velocity of projection.

16. A smooth inelastic particle slides along the sides of a regular polygon of n sides under the action of no forces; prove that its velocity after passing m angular points is diminished in the ratio of $\left(\cos\frac{2\pi}{n}\right)^m$ to 1.

17. Two masses m lbs. and m' lbs., moving with u velos and u' velos respectively impinge, so that u, u' make angles α, α' with the line of impact; prove that if after impact they are moving with v velos and v' velos respectively, then

$$mv^2 + m'v'^2 = mu^2 + m'u'^2 - \frac{mm'}{m+m'}(1-e^2)(u\cos\alpha - u'\cos\alpha')^2.$$

CHAPTER XI.

NEWTON'S LAWS OF MOTION.

134. Newton's Laws of Motion are

LEX I. *Corpus omne perseverare in statu quo quiescendi vel movendi uniformiter in directum, nisi quatenus illud a viribus impressis cogitur statum suum mutare.*

LEX II. *Mutationem motus proportionalem esse vi motrici impressæ, et fieri secundum lineam rectam qua vis illa imprimitur.*

LEX III. *Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigi.*

They may be translated as follows:

I. *Every body perseveres in its state of rest or of moving uniformly in a straight line, except in so far as it is made to change that state by external forces.*

II. *The change of the mass-velocity of a body is numerically equal to the impulse which produces it and is in the same direction.*

III. *To every action there corresponds an equal and opposite reaction; that is to say, the actions of two bodies upon each other are always equal and in opposite directions.*

Newton's Laws, I and II, follow immediately from Art. 31, provided, we define *Impulse* as the equivalent of a Force acting continuously for a definite interval; for force produces acceleration; therefore impulse produces additional velocity.

We may therefore in Art. 33 replace the words *force* and *acceleration* by the words *impulse* and *velocity*; and then Art. 33 is a statement of what is implied by Laws I and II.

Conversely, Art. 31 follows immediately from Laws I and II, provided we define *force* as, that which acting continuously for a definite interval is equivalent to an impulse; for impulse is that which produces additional velocity; therefore force must be that which produces acceleration.

Example. Consider a mass of 10 tons on a smooth horizontal plane (for instance a railway carriage on smooth horizontal rails), to which an horizontal impulse is applied by means of a hammer.

Suppose the mass of the hammer to be 2 lbs. and the velocity 32 velos; and suppose the hammer just brought to rest by the impact.

The impulse communicates the mass velocity 64 pound-velos to the 10 tons.

When the 10 tons is in motion, by making the hammer impinge again with a velocity 32 velos greater than the velocity of the 10 tons and bringing it relatively to rest by the impact, we can arrange that the impulse 64 pound-velos can be repeated any number of times.

Now suppose men employed to apply to the 10 tons in succession a series of impulses at intervals of $\frac{1}{100}$ of a second; *each* impulse communicating 64 pound-velos to the 10 tons, and suppose that they apply 100 impulses per second.

In such a case, if the force of each impulse lasts exactly $\frac{1}{100}$ of a second, and is uniform, then the 10 tons is acted on by a *continuous force*.

64 pound-velos are communicated to the 10 tons every $\frac{1}{100}$ of a second; that is, 6400 pound-velos per second, and

The motion is that produced by a force of 6400 poundals; or by the weight of about 200 lbs.

135. Let us suppose that there is such a thing as an *instantaneous* impulse.

Suppose an impulse ω pulses applied to a particle of unit mass (in the direction of the motion of the particle) at the beginning of each of a series of n intervals, each of τ seconds, such that $n\tau = t$.

The particle will receive at the beginning of each interval an additional velocity ω.

Next suppose the intervals diminished, their number increased, and the *ratio of the impulse to the interval* kept the same so that

$$\frac{\omega}{\tau} \text{ is constant.}$$

Let a stand for the ratio $\frac{\omega}{\tau}$.

Then in the limit, when the number n is infinitely increased, the motion of the particle is that of the uniformly increasing velocity of a celos which is caused by *continuous force.*

136. Hence an impulse which is equivalent to a force continuing for some given interval may be looked upon as a series of infinitely small impulses applied at infinitely small intervals whose sum is equal to the given interval.

The *magnitude* of a continuous force is measured by the *rate* at which it produces impulse.

137. It is not necessary however in the above to suppose the impulse *instantaneous.*

Let each impulse produce the velocity ω velos gradually and uniformly in the interval τ secs.; then, whether the intervals τ secs. are large or small, the motion of the particle is uniformly accelerated motion such as, by Art. 31, is produced by continuous force.

138. But just as the properties of curves are studied by supposing them to be derived from polygons, in which the sides are made to become infinitely numerous and infinitely short; so it is sometimes convenient to imagine our continuous force acting for a certain interval to be equivalent to a series of small impulses; these impulses being *instantaneous*, and becoming, in the limit, infinitely quick and infinitely small.

139. PROP. *To prove* the formula $s = ut + \frac{1}{2}at^2$ *of* Art. 21, *by* Newton's *method.*

Suppose a point to be moving in a straight line so that its velocity receives at the beginning of each one of a series of equal intervals of τ secs. an additional velocity ω velos.

Let n of these intervals of τ secs. make up t secs.

Let u be the velocity of the point at the beginning of the interval t seconds.

The velocity during the rth interval τ seconds is

$$(u + r\omega) \text{ velos.}$$

The distance passed over in the course of that interval is

$$(u + r\omega)\,\tau \text{ ft.}$$

So that the whole distance passed over in t seconds is

$$\{(u + \omega)\,\tau + (u + 2\omega)\,\tau + \ldots + (u + n\omega)\,\tau\} \text{ feet}$$

that is

$$\tfrac{1}{2}n\,\{2u\tau + (n + 1)\omega\tau\} \text{ feet}$$

that is

$$\{ut + \tfrac{1}{2}n\omega t + \tfrac{1}{2}\omega t\} \text{ feet. [Compare Art. 24.]}$$

Now suppose the interval τ seconds be diminished and consequently the number n increased so that $n\tau$ still equals t.

Suppose also that the *rate* at which velocity is added is kept unaltered, so that $\frac{\omega}{\tau}$ is a constant number $= a$; and

$$n\omega = \frac{n\tau\omega}{\tau} = \frac{t\omega}{\tau} = at.$$

Then the distance passed over in t seconds is

$$\left\{ut + \tfrac{1}{2}at^2 + \tfrac{1}{2}\,\frac{at^2}{n}\right\}.$$

This is true however great n may be; and hence it is true when n is greater than any assigned number; in which case the distance passed over in t seconds is

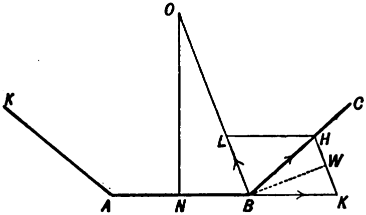
$$ut + \tfrac{1}{2}at^2.$$

140. The following proposition is an instructive example of Newton's method.

141. PROP. *A point moves with constant speed v* velos *along the perimeter of a regular polygon* ABC... *of* n *sides inscribed in a circle of radius r* ft.; *prove that at each angle the point receives an additional velocity towards the centre of the circle; and that the* rate of additional velocity per second *is* $\frac{v^2}{r}$.

Let O be the centre of the circle.

Let $AB = s$ ft.; let τ seconds be the interval occupied in going from A to B; then, $s = v\tau$.



Draw ON bisecting AB at right angles.

Produce AB to K so that BK represents the velocity v velos along AB;

let BH represent the velocity v velos along BC; then

$$BH = BK.$$

Complete the parallelogram $BKHL$.

Draw BW bisecting HK at right angles.

Then, since $BK = BH$, the angle $BKH = BHK = HBL$.

But the angle $ABL = BKH$;

therefore the angle $ABL = HBL$.

Therefore BL produced passes through the centre O.

Now the velocity represented by BH is the resultant of the velocities BK, BL.

That is, *the velocity* BL *when added to the velocity* BK *changes it into the velocity* BH.

In other words, when a point having v velos along AB

changes its velocity to v velos along BC, it receives an additional velocity ω velos represented by BL, whose direction is towards the centre of the circle.

The rate of additional velocity per second is $\frac{\omega}{\tau}$.

But, $$\frac{\omega}{\tau} = \frac{\omega v}{s} = \frac{v^2}{s} \times \frac{\omega}{v}$$

and, $$\frac{\omega}{v} = \frac{HK}{BK} = \frac{2KW}{BK} = \frac{2NB}{OB} = \frac{s}{r}.$$

Therefore, $$\frac{\omega}{\tau} = \frac{v^2}{s} \times \frac{\omega}{v} = \frac{v^2}{s} \times \frac{s}{r} = \frac{v^2}{r}.$$

That is, the required rate is $\frac{v^2}{r}$. Q. E. D.

142. PROP. *A point moves with the constant speed* v velos *along the circumference of a circle of* radius r; *prove that this point has constant acceleration* $\frac{v^2}{r}$ celos *towards the centre of the circle.*

We consider a circle to be the curve which a regular polygon approaches as its limit, when the number of its sides is infinitely increased.

When the polygon in Art. 141 becomes a circle the additional velocity at each angular point is infinitely diminished, the number of angular points being infinitely increased.

Hence, instead of a series of sudden increases of velocity we have, for the circle, a *continuous* growth of velocity, always in the direction of the centre, at the rate $\frac{v^2}{r}$ velos per second; that is, at the rate $\frac{v^2}{r}$.

N.B. Neither in the polygon nor in the circle does this continuous increase of velocity alter the *speed.* It alters the velocity as regards *direction* only.

143. PROP. *A particle of mass* m lbs. *is describing a circle of radius* r ft. *with* v velos; *prove that it is acted on by a force* $\frac{mv^2}{r}$ poundals *towards the centre.*

The particle has a constant acceleration $\frac{v^2}{r}$ celos towards the centre.

Acceleration never exists unless force causes it; hence there is a force $\frac{mv^2}{r}$ poundals towards the centre.

144. A force which always tends towards a fixed point is called a **centripetal** force.

Example i. *A particle of* 1 lb., *fastened to a fixed point* O *by a light string* 2 ft. long, *describes the circle about* O *as centre once every second; find the* tension *of the string.*

[NOTE. In a question like this, gravity is not supposed to be acting. The particle might be moving on a perfectly smooth horizontal plane. The student should notice that unless some word such as, *weight*, *horizontal*, *vertical*, *heavy*, appears in a question, gravity is supposed not to act.]

The particle describes the circumference of a circle of 2 ft. radius in 1 sec.; therefore it moves at the rate of 4π ft. per sec.

Therefore by Art. 142 its acceleration is $\frac{16\pi^2}{2}$ celos.

Therefore it is acted on by $8\pi^2$ poundals.

The tension of the string is therefore equal to the weight of about

$$\frac{8 \times 22 \times 22}{32 \times 49} \text{ pounds, nearly;}$$

that is, to the weight of 2·47 lbs. nearly.

NOTE. When a mass is describing a circle with uniform speed, its velocity is being changed (not in magnitude, but) in direction. By reason of its inertia this change cannot be made in the velocity of the mass by anything except by external force. We have shewn that in order to effect this change a force of a certain magnitude must continuously act on the particle *towards the centre* of the circle. The inertia of the mass continuously opposes this force; and used to be said to be *centrifugal*. This word *centrifugal* is very misleading; for the student should carefully notice that if the *centripetal* force ceases, the motion of the mass is along a *tangent:* therefore the tendency of the mass should be called **tangential**, not centrifugal.

EXAMPLES. XXXV.

In the first four of the following questions gravity is not acting.

1. A point is describing a circle of radius 5 ft. with uniform speed 10 velos, find its acceleration.

2. A mass of 4 lbs. is describing a circle of radius 5 ft. with uniform speed of 10 velos; what force is acting upon it?

3. A mass m lbs. fastened by a string to a fixed point is describing a circle uniformly with v velos; the tension of the string is p poundals, find the length of the string.

4. A mass m lbs. is fastened by a string of length r feet to a fixed point O, and describes a circle about O n times per second; shew that the tension of the string is $2mn^2\pi^2r$ poundals.

5. A railway carriage is describing a curve on horizontal rails; shew that if it goes fast enough it must fall over.

6. A railway carriage of 10 tons is describing an arc of a circle of $\frac{1}{4}$ mile radius with a speed of 30 miles an hour; find the horizontal force which must be acting on the carriage.

7. A mass of 1 cwt. is in a railway truck which is moving, on horizontal rails on a curve whose radius is 400 yards, at the rate of 60 miles an hour; what is the horizontal stress between 1 cwt. and the truck?

8. A mass of 56 lbs. is placed in a swing the ropes of which are 10 ft. long; find the difference in the tension of the ropes when the swing is at its lowest point, (i) when it is at rest, (ii) when it is moving with 20 velos (neglecting the mass of the swing itself).

9. A horse weighing 5 cwt. is going along a road at the rate of 10 miles an hour and passes over a depression in the road whose section is that of a circle of 40 feet radius; what is the difference between his weight and the average pressure of his feet on the road when at the lowest point of the depression?

10. Two masses of 3 lbs. and 4 lbs. on a smooth horizontal plane are fastened together by a light inelastic string 7 ft. long and are each describing a circle so that one point in the string is at rest on the plane; what must be the point in the string?

11. Two masses A and B of 1 lb. each are joined by a string; the mass A describes a circle of radius 2 ft. with uniform speed on the surface of a smooth horizontal table, while the other mass B is suspended under the action of gravity by the string, which passes through a small hole in the table at the centre of the circle; find the speed of A.

12. Two masses A and B are joined by a string; the mass A of 10 lbs. describes on a smooth horizontal table a circle of radius 5 ft. with uniform speed 5 velos; the other mass B is suspended under the action of gravity by the string which passes through a small hole in the table at the centre of A's circle; find the mass of B that it may rest in equilibrium.

13. A particle of 3 lbs. is fastened to a fixed point by a string $2\sqrt{2}$ feet long and is describing a horizontal circle under the action of gravity, so that the string describes a right circular cone, the inclination of the string to the vertical being 45^0; prove that the velocity of the particle is about 8 velos.

14. Shew that the tension of the string in Question **13** is $3\sqrt{2}$ lbs. weight.

15. A particle of m lbs. is fastened to a fixed point by a string l feet long, and is describing a horizontal circle under the action of gravity, so that the string describes a right circular cone of vertical angle 2α; prove that the velocity of the particle is

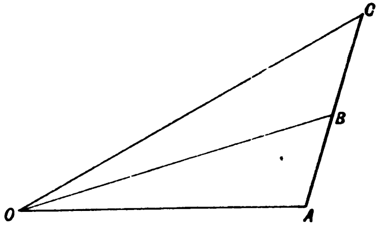
$$\sqrt{(lg \sin^2 \alpha \sec \alpha)} \text{ velos.}$$

NOTE. The arrangement described in Question **15** is called a *Conical Pendulum.*

16. One end of a string $2l$ feet long is fastened to a point A on a fixed smooth vertical rod, the other to a small ring P of mass m lbs. which slides on the rod; another mass Q m' lbs. is fastened to the middle point of the string and revolves with velocity v velos in a horizontal circle so that the angle AQP is a right angle, prove that

$$v^2 = \frac{lg}{\sqrt{2}} \frac{(m' + 2m)}{m}.$$

145. PROP. *When a point* P *is moving with uniform velocity, the line* OP *joining* P *to any fixed point* O, *traces out equal areas in equal intervals.*



Let the point P be moving in the straight line ABD with uniform velocity; P passes over equal distances in equal intervals; let AB and BD be two distances passed over by P in any two equal intervals; then, $AB = BD$.

Therefore the area $OAB = OBD$.

But the areas OAB, OBD are areas traced out by OP in any two equal intervals; so that OP traces out equal areas in equal intervals. Q. E. D.

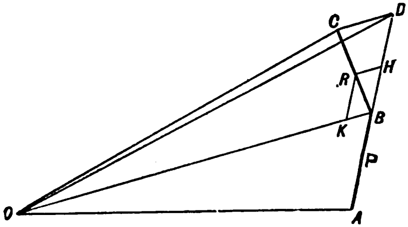
146. PROP. *A point* P *is moving with uniform velocity* u *velos, (so that the line* OP *joining* P *to a fixed point* O *is tracing out equal areas in equal intervals); at a certain instant the velocity of* P *receives a certain addition of* ω *velos, in the direction* PO; *it is required to prove that such an addition of velocity does not alter the rate at which* OP *is tracing out area.*

Let the point P be moving with u velos along AB.

Produce AB to D; let BH represent u velos.

When P is at B let it receive an additional velocity ω velos represented by BK, where BK produced passes through O.

Complete the parallelogram $BKRH$; then the subsequent path of P is along BR, with velocity v velos represented by BR.



Let $AB = ut$ ft.; produce AB to D, making $BD = AB$; through D draw DC parallel to BK; then $BC = vt$ ft.; so that AB and BC are the distances passed over by P in equal intervals of t seconds, just before and just after, the addition of the velocity ω velos. Also AOB and BOC are the areas traced out in the same equal intervals.

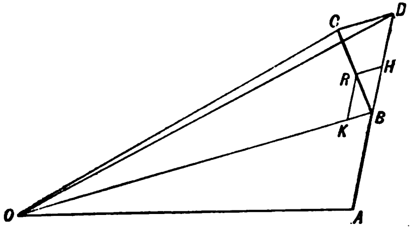
But the area $BOC = BOD$, since CD is parallel to OB; and the area $BOD = AOB$, for $AB = BD$.

Therefore, the area $BOC = AOB$.

That is, the area traced out by OP in any two equal intervals just before and just after the addition of the velocity BK, is the same.

In other words, the rate at which OP traces out area is unchanged. Q. E. D.

147. PROP. The converse of Art. 146. *If a point* P, *moving with uniform velocity* u *velos along* AB, *receive an additional velocity when it is at* B, *in such a manner that the* rate, *at which the line joining* P *to some fixed point* O *traces out* area, *is unchanged, then this additional velocity must be in the direction* BO.



Let AOB and BOC be the areas traced out by OP in equal intervals of t seconds. Produce AB to D, making $BD = AB$.

Let BH represent u velos; let BR represent the resultant velocity; then HR represents the additional velocity; and since

$$BD = AB = ut \text{ and } BC = vt,$$

HR is parallel to DC.

But by hypothesis, the area $OBC = AOB$, and the area $AOB = OBD$. Therefore the area $OBC = OBD$.

Wherefore DC is parallel to BO.

But the additional velocity at B is parallel to DC and therefore is in the direction BO. Q. E. D.

148. It follows from the proposition of Arts. 146 and 147, that when the velocity of a moving point P receives any *series* of additions each of which is such that it is in the direction of the line joining P at that instant to a fixed point O, then the line OP traces out equal areas in equal intervals.

149. And conversely, if the line joining a moving point to a fixed point O traces out areas at a constant rate per second, then whatever addition is made to its velocity that addition must always be in the direction which PO has at that instant.

150. This is true whether the additional velocities are large or small and whether the intervals are large or small.

151. Suppose now that a force of any magnitude acts on a particle P of mass m, so that the force is always in the direction of the line joining P to a fixed point O.

Newton imagines this force to be the limit of a series of small instantaneous impulses acting on the particle at small intervals. These impulses will each be along the line joining P to O at the instant of its action. Accordingly these impulses will generate small additional velocities in the particle, each velocity being *in the direction PO.*

Therefore however small and however frequent these impulses are, the rate at which OP traces out area will be unaltered by them.

Hence, when a particle P is acted on by a *force* which always is directed towards a fixed point O, OP traces out equal areas in equal intervals.

152. The converse is also true. If the motion of a particle P is such, that the line OP joining P to a fixed point O, traces out equal areas in equal intervals, then whatever force acts upon P, the direction of this force must always be towards O.

EXAMPLES. XXXVI.

1. Prove that a mass which is describing a circle with uniform speed, is acted on by a force whose direction tends always towards the centre of the circle.

2. Kepler observed that the line joining the centre of the sun S to any one of the planets P, traces out equal areas in equal intervals; what is the direction of the force under which the planets are moving?

3. Prove that a Planet moves faster when nearest the Sun than when it is furthest from the sun.

CHAPTER XII.

RELATIVE MOTION.

153. When we speak of the *distance* of a point, we must always fix upon another point *from which* that distance is to be measured.

So when we speak of the *velocity* of a point P (that is, of the rate of increase of the distance of the point P), we must again fix upon some point O *from which* the distance which increases is to be measured.

Thus the velocity of a point is a *relative* term only.

154. When the point O, from which the distance of P is measured, is at a distance from another point O', then the distance of P from O' is the resultant of the distances of P from O and of O from O'.

Similarly, when P has velocity relatively to O, and O a velocity relatively to O', then the velocity of P relatively to O' is the resultant of those two velocities.

For example, when we speak of the velocity of a railway train we mean its velocity relative to some point on the earth's surface. The surface of the earth is itself in rapid motion.

155. When a number of points have, besides their own proper motion, a velocity *common* to each of them, this common velocity has no effect on the *relative* distances, after any interval, of the points *from each other*.

156. The same is true of an *acceleration common* to a number of points.

157. Therefore it is also true, when a *force* is applied to each of a system of particles, such that the magnitude of each force is proportional to the mass of the particle to which it is applied (so that each particle has produced in it a certain acceleration).

For example, the *weight* of any particle produces in it a certain fixed acceleration.

Example. A small shell explodes in mid-air, in such a way that each fragment has the same additional initial velocity u velos *away from the centre of the shell, given to it at the instant of explosion; shew that (neglecting the resistance of the air) after any interval* t secs. *the fragments all lie in the circumference of a* sphere of radius *ut* ft.

Each fragment is moving (i) with the velocity of the shell, say v velos at the instant of the explosion, (ii) with the additional velocity u velos, and (iii) with the acceleration g celos due to gravity, downwards.

To find its position after t seconds we must draw from the initial position O (which is that of the shell at the instant of explosion), a line vt ft. parallel to v, a line $\frac{1}{2}gt^2$ vertically downwards, arriving at a point O'; then from O' a line ut in a different fixed direction for each fragment.

The point O' is the same for each fragment; so that after t seconds *each* fragment is at a distance ut ft. from the same point O'. This point O' is the position which the small shell would have occupied had it not exploded.

EXAMPLES. XXXVII.

1. A shell when moving horizontally explodes so that its mass is equally divided and one part is brought momentarily to rest by the explosion; shew that the velocity of the other part is doubled by the explosion; and if the shell was at the height of 400 ft. and moving with 100 velos, shew that the distance between the two portions on reaching the ground is 1000 feet.

2. A number of guns are fired simultaneously in different directions each making the same angle with the horizon, the muzzle velocity of the bullets being the same; prove that the bullets are after any interval to be found on the circumference of a horizontal circle.

3. A number of rifles are simultaneously discharged in all directions from the same point, the bullets having each a muzzle velocity of 1000 velos; prove that when the bullet which was projected vertically has reached its highest point, all the bullets are on the surface of a sphere of radius 31250 ft.

4. A number of particles are simultaneously swept off a table with different horizontal velocities; shew that they all reach the ground simultaneously.

5. A series of particles are simultaneously projected along a horizontal line on an inclined plane with different horizontal velocities; shew that as they slide down the plane they continue in the same horizontal line.

6. Shew that if a series of spheres are projected as in Question **5**, which impinge on one another, they will always be in the same horizontal line.

7. Prove that the force which a railway engine exerts in pulling a train, over and above that necessary to overcome the friction, is distributed among the component parts of the train in the proportion of their masses.

8. Shew that if any number of particles are moving with the same velocity in space, and parallel forces proportional to the masses of these particles are applied to each of them respectively, the relative positions of the particles will not be altered thereby.

CHAPTER XIII.

THE HODOGRAPH.

158. We represent the path of a moving point P by a line drawn on the paper.

The place from which the point is observed is represented on the paper by a fixed point O; the distance of P from O is represented by the straight line OP.

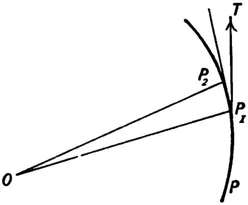
Consider the positions P_1, P_2 which P has at the beginning and at the end of an interval t secs.

The additional distance of P in the interval t secs. is represented by the straight line P_1P_2.

When P is moving with uniform velocity u velos, then $P_1P_2 = ut$ ft.

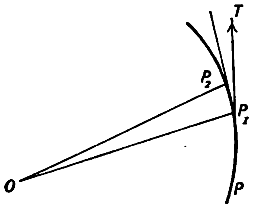
159. When the path of P is a curved line, a tangent to the curve at a point P_1, gives the *direction* of the velocity of P at the instant when the point is at P_1.

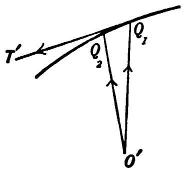
Take a point P_2 near to P_1.



Then P_1P_2 is a line whose direction approaches that of the tangent at P_1, when P_2 is made to move nearer to P_1.

Also, if $P_1P_2 = s$ ft., and the point P take t secs. in going from P_1 to P_2, then $\frac{s}{t}$ approaches to the number of velos in the velocity of P when at P_1, as P_2 approaches P_1.





160. Now take another fixed point O'; from O' draw a line $O'Q$ to represent the *velocity* of the moving point P. If we consider Q to move so that the line $O'Q$ is always parallel and proportional to the velocity of P, Q will trace out a line. The line traced in this way by the point Q is called the **Hodograph** of the point P.

161. On the hodograph take the two points Q_1, Q_2 which correspond to the points P_1, P_2 on the path of P. Then Q_1Q_2 represents the additional velocity added to P in the interval t secs.

For OQ_2 is the velocity at P_2, and it is the resultant of the two velocities OQ_1 and Q_1Q_2. [Art. 99.]

162. Suppose Q_1 Q_2 represent v velos, then, as P_2 approaches P_1, the ratio $\frac{v}{t}$ approaches the number of celos in the acceleration which P has when at P_1.

Also, the direction of the tangent to the hodograph at Q_1, is the direction of the acceleration of P when at P_1.

In other words, the velocity of Q in the hodograph represents the acceleration of P on the curve.

Example i. The hodograph of a point which moves with uniform velocity is *a point*.

The line OQ is fixed.

Example ii. When a point is moving with a velocity which is uniformly increasing in a given direction, the hodograph is a straight line along which it moves with uniform velocity.

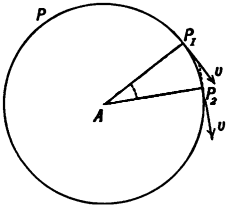
Example iii. When a point is moving with constant *speed* [See Art. 78] in the circumference of a circle, its hodograph is a circle; and the point Q moves along the hodograph with uniform *speed.*

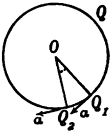
For OQ represents the velocity of P, and it is of constant magnitude and its direction turns through equal angles in equal intervals.

Example iv. *When a point moves with constant speed* v velos *along the circumference of a circle of radius* r feet, *it has a constant acceleration* $\frac{\mathrm{v}^2}{\mathrm{r}}$ celos *whose direction always points to the centre of the circle.*

Let the point P be describing the circle PP_1 whose centre is A with constant speed.

Draw the hodograph of P; let this be the circle QQ_1 whose centre is O; then the acceleration of P is represented by the velocity of Q.





Let Q_1, Q_2 be points on the hodograph corresponding to the points P_1, P_2 on the path of P.

Then OQ_1 is parallel to the velocity of P at P_1 and is therefore perpendicular to AP_1.

The velocity of Q at Q_1 is perpendicular to OQ_1, which is parallel to P_1A and is constant. Therefore the acceleration of P at P_1 is in the direction P_1A and is constant.

Now Q describes the circumference of its circle while P describes the circumference of its circle; let the interval in which this is done be t secs.; let the velocity of Q be a velos.

Then, $O'Q = v$ ft. and $at = 2\pi v$,

also, $vt = 2\pi r$,

whence, $a = \frac{v^2}{r}$. Q. E. D.

SECTION IV.

WORK, ENERGY, POWER.

CHAPTER XIV.

WORK.

163. An external force is said to be doing **work** on a mass when its point of application is passing over *distance* in the direction of the force.

Examples. The weight of a mass falling freely does work on the mass.

The pressure of the foot of a bicyclist does work on the treadle of his bicycle.

The pull of a horse does work on the carriage it is drawing.

When a horse is attached to a railway truck and pulls at it, then as long as the truck is *at rest*, we say the horse is *leaning* against the truck, and does *no work*; but when the truck *moves* and the horse continues his pull, he does work as long as his pull is continued.

164. DEF. The **work** done in an interval on a mass by an external force, varies directly as the force; and also directly as the resolved part in the direction of the force, of the *distance* passed over in that interval by the point of application of the force.

Thus, the work done by a force applied to a mass in motion, is doubled, trebled, etc., when the force is doubled, trebled, etc., for the same distance; and the work done by a given force is doubled, trebled, etc., when the resolved part of the distance in the direction of the force is doubled, trebled, etc.

It is convenient to refer to the resolved part in the direction of the force, of the distance passed over by the point of application of a force, as *the distance through which the force works.*

NOTE. The work done by an external force on a moving mass is *independent of the length of the interval occupied;* it depends only on the *force* and the *distance.*

165. DEF. We shall choose as our **unit** *work* the *work* done by a *poundal* while its point of application moves over a *foot* in the direction of the poundal.

We shall call this *unit* work a **foot-poundal.**

Hence the **measure** of the work done by a uniform force, is the product of the *number* of poundals by the *number* of feet through which it works.

Example. The work done by the *weight* of 1 lb. while the 1 lb. passes over 1 foot vertically downwards, is g foot-poundals.

For, the weight of 1 lb. $= g$ poundals,

1 poundal acting through 1 ft. = 1 foot-poundal,

g poundals acting through 1 ft. $= g$ foot-poundals.

166. When the distance through which the force works, is passed over by the point of application in the direction *opposite* to that of the force, this distance is *negative;* and in this case the work done by the force is said to be **negative**; or we say that work is done *against* the force.

Work which is done in lifting mass under the action of gravity, is done *against* the weight of the mass.

167. The work done against gravity in raising 1 lb. through a vertical distance of 1 foot is g foot-poundals.

This amount of work is called a **foot-pound**, which is an abbreviation for a **foot-poundweight.**

Hence a *foot-pound* is equal to g *foot-poundals.*

Example. Find the work done *by the weight of* 1 cwt. *while the* cwt. *falls a vertical distance of* 10 feet.

The force is 32×112 poundals.

The distance passed over by the mass in the direction of the force, that is, the distance through which the weight works, is 10 ft.

The work done is therefore $\underline{32 \times 112 \times 10 \text{ foot-poundals.}}$

168. When two equal and opposite forces (as, for example, the action and reaction of a stress) act upon the *same* point the work done by one of the forces is equal and opposite to the work done by the other.

Also, if two equal and opposite forces act upon different points which have no motion relative to each other, the work done by one of the forces is equal and opposite to that done by the other force.

The total work done by two such forces is zero.

Example i. The tension of an inextensible string does no work.

A man in a railway carriage who presses with his feet against the opposite seat does no work, notwithstanding that the mass, to which he applies two equal and opposite forces, is in motion.

Example ii. When the masses in an Atwood's machine are equal and are in motion the weight of one does as much work on its mass as the second mass does against its own weight.

Example iii. 1 cwt. *slides down an inclined plane of height* 10 ft. *and inclination* α; *find the* work done *by the* weight *of the mass when the mass goes from the top to the bottom.*

The distance passed over by the mass is the slant side of the plane; the direction of the force is vertically downward; the resolved part of the distance in the direction of the force is the vertical distance passed over; this is, the height of the plane, viz. 10 feet.

Hence, the work done is $10 \times 112 \times 32$ foot-poundals;

that is, 1120 foot-pounds.

Example iv. *A horse draws a tram-car of* 2 tons *along a horizontal road with the uniform velocity of* $7\frac{1}{2}$ miles an hour; *supposing the resistance of the friction etc. is* $\frac{1}{80}$ *of the weight of the car, find how many* foot-pounds *the horse is doing* per second.

The resistance of friction etc. is a force equal to the weight of $\frac{1}{80}$ of 2 tons; that is, the weight of 56 lbs.

Therefore the horse must be applying a force of $56 \times g$ poundals; [for, since the velocity of the car is uniform, no additional force is required by the inertia of the car.]

The point of application of this force is moving at the rate of $7\frac{1}{2}$ miles per hour; that is, of 11 velos.

Therefore the horse is doing $11 \times 56 \times g$ foot-poundals per second;

that is, 616 foot-pounds per second.

169. PROP: *The number of* foot-pounds in the work done *by the* weight of *a mass while the mass is moved from one position to another is the product of the* number of lbs. weight *by the* number of feet *in the vertical distance of the second position below the first.*

For we may consider the path of the mass to be along a number of inclined planes; the work done by the weight in descending any one of these planes is the same as the work done in descending the vertical height [*Example* iii. above].

The work done in *ascending* any inclined plane is that due to its vertical height and is negative; hence

The work done by the weight of a mass in going by any path whatever from one position to another is the same as that done by the weight of the mass in descending from the first position to the horizontal plane in which the second position lies.

Thus, the work done in raising 1 ton through a vertical distance 50 feet is the same, viz. $2240 \times 50 \times g$ foot-poundals, by whatever path the ton arrives at that height. It may be raised by screws; or by a system of pulleys; or by a horse pulling it up an incline in a cart; or it may be raised by a man carrying it piece-meal up a ladder.

EXAMPLES. XXXVIII.

Find the number of foot-poundals and of foot-pounds done by the weights of the following.

1. 56 lbs. in descending 40 feet.

2. A man of 10 stone in descending 100 feet.

3. A block of stone of 10 tons in descending 500 feet.

4. A man of 12 stone in ascending a tower 200 ft. high.

5. A train of 300 tons in descending an incline of 1 in 50, one mile in length.

6. A waggon and horses of 3 tons in descending a hill, whose inclination is 1 in 30, for a distance of 200 yards.

7. Compare the work done in 1 hour against gravity (i) by a man who lifts 28 lbs. a height of 3 ft. every minute, (ii) by a man of 10 stone who walks up an incline of 1 in 6 at the rate of 4 miles an hour.

8. Find the work done against friction in moving a ton 100 feet along a rough horizontal plane the horizontal resistance of the friction being $\frac{1}{10}$ of the weight.

9. Find the number of foot-pounds per second required to draw a cart of 1 ton along a horizonal plane uniformly at the rate of 5 miles an hour, the resistance due to friction etc. being $\frac{1}{70}$ of the weight.

10. Find the greatest uniform velocity with which an engine capable of producing 140,000 foot-pounds of work per second, can draw a train of 100 tons along a horizontal plane, the resistance due to friction etc. being $\frac{1}{80}$ of the weight.

11. An engine is drawing a train of 120 tons up a smooth inclined plane of 1 in 60 at the rate of 24 miles an hour; how much work is it doing per second (friction to be neglected)?

12. Supposing the resistance due to friction etc. in Question **11** to be equivalent to the weight of 1120 lbs. and to act opposite to the direction of motion, find at what rate the engine could draw the same train up an incline of 1 in 60, doing the same amount of work per second as before.

13. Supposing the resistance due to friction in Question **12** to be $\frac{1}{240}$th of the *pressure* of the train on the inclined plane, find at what rate the engine could draw the train up the inclined plane with the same expenditure of work per second.

14. Supposing that a man of 12 stone in walking raises his whole weight a vertical distance of 1 inch at every step, and that the length of each step is $2\frac{1}{2}$ feet, find how much work the man does in this way in walking 1 mile.

15. Compare the work done by the man in Question **14** with that done by another man of 12 stone going a mile with uniform speed on a bicycle weighing $\frac{1}{2}$ cwt., the resistance due to friction etc. of the bicycle being $\frac{1}{100}$ of the weight.

CHAPTER XV.

ENERGY.

170. DEF. **Energy** is *capacity for doing work.*

There are many different forms of energy, of which two chiefly concern us in Dynamics; viz. *Potential Energy* and *Kinetic Energy.*

171. DEF. **Potential** *energy* is capacity for doing work which a mass has by virtue of its *position.*

A spring when *bent*, has energy by reason of its position; for it can do work when changing that position.

Air when compressed can do work on being allowed to expand.

A mass at the top of a tower can do work in its descent to a lower level.

Example. Find the difference between the Potential Energy *of* 1 cwt. *at the* top *of a tower* 100 ft. *high, and that of* 1 cwt. *at the foot of the tower.*

In descending from the top of the tower to the bottom, the *weight* of the 1 cwt. does 100 × 112 foot-pounds;

Hence, 1 cwt. at the top of a tower 100 ft. high, has 100 × 112 foot-pounds more Potential Energy than 1 cwt. at the foot of the tower.

172. DEF. **Kinetic** *energy* is the capacity for doing work which a mass has by virtue of its mass-velocity.

A bullet when in motion can do work in giving up its velocity.

Water when in motion can do work in turning a water wheel.

The mass of a hammer can do work by virtue of its velocity.

The kinetic energy of a particle having a given velocity is measured by the number of foot-poundals it does in giving up that velocity.

173. PROP. *Prove that the difference between the energy of* m lbs. *having* v velos, *and that of* m lbs. *having* u velos, *is* $(\frac{1}{2}mv^2 - \frac{1}{2}mu^2)$ foot-poundals.

Let a force of p poundals be applied to the m lbs. so that in t seconds its velocity is reduced from v velos to u velos; also suppose that in that interval the mass has a retardation a and passes over s feet in the direction opposite to the force; then, by Art. 26. iii,

$$as = \tfrac{1}{2}v^2 - \tfrac{1}{2}u^2,$$

or,

$$mas = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2;$$

but ma is the number of poundals required to produce in m lbs. acceleration a; so that $ma = p$.

Therefore $$ps = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2.$$

But ps is the number of foot-poundals done by p poundals which acts on a mass while it passes over s feet in its own direction. That is to say, the mass m lbs. having velocity v can do $(\frac{1}{2}mv^2 - \frac{1}{2}mu^2)$ foot-poundals while its velocity is reduced from v to u. Q.E.D. Hence

174. The kinetic energy of a particle of m lbs. which has v velos is $\frac{1}{2}mv^2$ foot-poundals.

For the mass can do $\frac{1}{2}mv^2$ foot-poundals against a force before it is reduced to rest.

Also, if $\frac{1}{2}mv^2$ foot-poundals of work are done on a particle of mass m lbs. at rest, it will have v velos.

Example. Find the work done by the engine of a train of 120 tons, *moving the train a* mile *from rest and at the same time giving it a speed of* 30 miles per hour; *the motion being on a horizontal line and the resistance due to friction* $\frac{1}{240}$ of the weight of the train.

The work done against friction is

$$(\tfrac{1}{240} \times 2240 \times 120) \times (1760 \times 3) \text{ foot-pounds,}$$
$$= 1120 \times 1760 \times 3 \text{ foot-pounds.}$$

The work done on the inertia, in producing kinetic energy, is

$$\tfrac{1}{2} \times 120 \times 2240 \times (44)^2 \text{ foot-poundals} \qquad \text{[Art. 173.]}$$

$$= \tfrac{1}{32} \times \tfrac{1}{2} \times 120 \times 2240 \times (44)^2 \text{ foot-pounds.}$$

Hence, the whole work done by the engine is

$$(1120 \times 30 \times 176 + 1120 \times 30 \times 2 \times 121) \text{ foot-pounds}$$

$$= \underline{1120 \times 30 \times 418 \text{ foot-pounds.}}$$

EXAMPLES. XXXIX.

Find the kinetic energy of each of the six following;

1. A stone of 4 lbs. which has 200 velos.

2. A cannon shot of 8 cwt. which has 2000 velos.

3. A cannon shot of 12 cwt. which has 1600 velos.

4. A train of 400 tons going at the rate of 60 miles an hour.

5. A man of 12 stone running at the rate of 12 miles an hour.

6. A man of 12 stone after falling a distance of 10 feet.

7. Find the work done by the engine of a train of 360 tons, in going 2 miles from rest on horizontal rails, the friction being $\frac{1}{240}$ of the weight, and the speed attained 45 miles per hour.

8. Find the work necessary to raise a train of 100 tons to the top of a mountain pass 500 feet high and at the same time to give it a velocity of 60 miles an hour (neglecting friction etc.).

9. A man has to raise 1 cwt. of bricks 8 feet; he throws them up so that they arrive at a point 8 ft. high with 5 velos; compare the superfluous work done with the necessary work.

10. A tram car of 1 ton is stopped by a brake 10 times in going a mile; the brake stops the car in 11 yards; after each stoppage the car attains a velocity of 7½ miles an hour. Supposing the friction of the rails to be a uniform force of 28 lbs. weight, compare the work done in this journey with the work done in going a mile with uniform velocity.

11. The expense of moving a train is proportional to the work done. Compare the cost of getting the speed of a train up from rest to a velocity of 45 miles an hour and at the same time going 1 mile, with the cost of moving it a mile with uniform velocity; supposing that the resistance caused by friction etc. to be $\frac{1}{120}$ of the weight of the train.

12. Suppose the tram-car in Question 9 had no brake, and so had to be stopped by the horses; find how much more work the horses would have to do in consequence in a journey which includes 10 stoppages.

175. When a particle slides along a *smooth* **plane** the pressure of the plane, whatever it is, is always perpendicular to the plane, and therefore perpendicular to the path of the particle; the *distance* through which the pressure *works* is therefore zero; hence

176. The pressure of a smooth plane on a particle moving along its surface does no work on the particle.

Example. When a particle slides down a smooth inclined plane under the action of gravity only, its total energy *is unchanged.*

By *Example* i. p. 91, a particle of m lbs. sliding down an inclined plane of angle a has $g \sin a$ celos.

Let h feet be the *vertical* distance passed over by the particle in any interval t seconds during which the velocity of the particle is changed from u velos to v velos.

Then the particle has lost mgh foot-poundals of potential energy and gained $(\frac{1}{2}mv^2 - \frac{1}{2}mu^2)$ foot-poundals of kinetic energy; we have to prove that these amounts of work are equal.

Let s feet be the distance passed over on the plane,

then $$s\,(g \sin a) = \tfrac{1}{2}v^2 - \tfrac{1}{2}u^2,$$

but $$h = s \sin a.$$

Therefore $$hg = \tfrac{1}{2}v^2 - \tfrac{1}{2}u^2.$$ Q. E. D.

177. The sum of the potential energy and of the kinetic energy of a particle is unaltered by any motion under the action of gravity only.

Example i. Let m lbs. fall from rest h feet *vertically*. Then the potential energy is diminished by mh foot-pounds.

It receives v velos, where $mgh = \frac{1}{2}mv^2$; so that it has received kinetic energy capable of doing mgh foot-poundals, that is, mh foot-pounds; in other words, the potential energy lost is equal to the kinetic energy gained.

Example ii. Since the velocity of a projectile is equal to that due to a fall from the directrix, it follows that (the kinetic energy + the potential energy) of a projectile is always equal to the potential energy of an equal particle at rest at some point on the directrix.

178. When a particle slides along a **curved surface** which is *smooth*, it is understood that the pressure of the surface on the particle is always perpendicular to the surface, and therefore this pressure is *always* perpendicular to the path of the particle.

The *distance* through which the pressure works is therefore *always* zero; so that the *work* done by the pressure is zero; hence,

179. *When a particle is sliding on a smooth surface under the action of no forces its kinetic energy is constant;* that is, *its speed is constant.*

*180. We may also prove this important proposition as follows: [See also Ex. **21**, p. 144.]

PROP. *An inelastic particle slides along the smooth sides of a regular polygon; prove that, in the limit when the polygon becomes a circle, the impacts at the angular points have no effect on its speed.*

Let AB, BC be two of the sides of the regular polygon. Produce AB to K.

Let the velocity along AB be v velos.

Resolve this velocity along and perpendicular to BC; the resolved parts are $v \cos \alpha$ velos and $v \sin \alpha$ velos.

The resolved part $v \sin \alpha$ velos is destroyed by the impact.

Hence after impact the particle will have $v \cos \alpha$ velos along BC.

Similarly after the next impact it will have $(v \cos \alpha) \cos \alpha$ velos along the next side and so on.

Hence, when the velocity of the particle is turned through a finite angle A, where $n\alpha = A$, it will have $v \cos^n \alpha$ velos,

that is $$v \left(\cos \frac{A}{n}\right)^n \text{velos.}$$

Now let n be increased and α diminished without limit so that A is kept unchanged; then the limit of $\left(\cos\frac{A}{n}\right)^n$ is 1.†

Hence, when the polygon becomes a circle, $v(\cos\alpha)^n = v$.

So that the speed of the particle is unchanged, when its direction has turned through a finite angle A. Q. E. D.

*181. Since the above proposition is true of a circle of *any* radius, and of an arc of any magnitude, it must be true for *any* continuous curve.

For such a curve may be considered to be made up of successive small arcs of its successive circles of curvature.

182. When a particle slides on a smooth continuously curved surface, under the action of gravity only, its total energy is constant.

For, the only forces acting are the pressure of the surface and the weight of the particle; and the pressure of the surface does not alter the kinetic energy of the particle, while the weight of the particle produces its own effect, no matter what other forces may be acting. The effect of the weight is expressed by saying that the total energy of the particle is constant.

Example 1. *A smooth fine wire, in the form of a circle of radius* r ft. *is placed in a vertical position, and a particle in the form of a small ring slides on it under the action of gravity. The ring has* u velos *when at the highest point of the circle; find its* velocity at the lowest point.

Let the ring contain m lbs.

In going from the highest point to the lowest it loses

$2\,rmg$ foot-poundals of potential energy.

Let its velocity at the lowest point be v velos; it will therefore have gained $(\frac{1}{2}mv^2 - \frac{1}{2}mu^2)$ foot-poundals of kinetic energy.

But on the whole, its energy is unchanged.

Therefore, $$2\,rmg = (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2),$$

or, $$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 + 2\,rmg.$$

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† Lock's Higher Trigonometry, Art. 9.

Example ii. *A stone is fastened by a light inextensible string* 1 foot *long, to a fixed point, and is describing a vertical circle about the fixed point as centre, under the action of gravity; find the* least possible velocity at the lowest point of the circle.

The constraint of the string is similar to that of a smooth circular wire; for the tension of the string, being always perpendicular to the direction of motion, does no work; hence, by Art. 179, the total energy of the stone is constant.

Therefore, if v velos be the velocity of the stone at the lowest point of the circle, u velos its velocity at a point on the circle whose vertical distance above the lowest point is h feet, then

$$mgh = (\tfrac{1}{2}\, mv^2 - \tfrac{1}{2} mu^2)$$

or, $$\tfrac{1}{2}\, v^2 - \tfrac{1}{2}\, u^2 = gh \text{..............................(i).}$$

Now, in order that the stone may completely describe the circle, the tension of the string (which is *least* at the highest point) must just vanish at that highest point.

Let u velos be the velocity of the particle at the highest point, then its acceleration is

$$\frac{u^2}{r} \text{ celos vertically downwards.}$$

The only force acting when the stone is at the highest point of the circle, is the weight, mg poundals vertically downwards; for the only other force available is the tension of the string, which at this moment is zero.

Hence, $$mg = \frac{mu^2}{r}$$

$$= mu^2 \text{ [for } r = 1\text{] ....(ii).}$$

Therefore, since v is the velocity at the lowest point of the circle, putting $h = 2$ ft. in (i) and combining with (ii), we have

$$v^2 = u^2 + 4g = g + 4g = 5g.$$

Hence, the least velocity at the highest point is about $4\sqrt{2}$ velos, and the corresponding velocity at the lowest point about $4\sqrt{10}$ velos.

NOTE. The unit of work in the C.G.S. system of units [see page 65] is the work done by a Dyne working through a Centimetre.

This unit work is called an **Erg.**

EXAMPLES. XL.

1. A particle is describing a parabola under the action of gravity, prove that its total energy is constant.

2. A heavy particle slides down a smooth straight tube from rest and then describes a parabola; find its velocity when it is 100 feet below the point from which it started.

3. A perfectly elastic particle is let fall on a horizontal pavement, and rebounds; prove that the total energy of the particle is constant.

4. Two perfectly elastic spheres impinge directly, prove that their total energy before and after impact is the same.

5. Two perfectly elastic spheres impinge obliquely, prove that their total energy is unaltered by the impact.

6. An imperfectly elastic particle falls on a horizontal plane and rebounds; shew that it loses energy by the impact.

7. Two imperfectly elastic balls m and m', of elasticity e and velocities u velos, u' velos impinge directly; find the energy lost by impact.

8. A multitude of small perfectly elastic spheres are in motion in a closed space surrounded by fixed walls, under the action of no force; prove that the total kinetic energy of the spheres is constant.

9. If the spheres in Question **8** are under the action of gravity, shew that their total energy is constant.

NOTE. A **simple pendulum** consists of a heavy particle fastened to a fixed point by a light rod; the particle moves in the lowest arc of a vertical circle under the action of gravity.

10. A simple pendulum *of length* l feet swings through an arc 4α, prove that its velocity at the lowest point of its path is $2\sin\alpha\sqrt{(gl)}$ velos.

11. A circular wheel, whose rim consists of a heavy uniform wire of m lbs. and whose spokes, (of length r feet), and axle are such that their mass may be neglected, revolves about its axis, which is fixed, so as to make n complete revolutions per second; shew that the kinetic energy of the wheel is $2mn^2\pi^2r^2$ foot poundals.

12. A fine light string is coiled round the wheel of Question **11**, one end being fastened to the rim and the other end supporting a heavy mass m' lbs. This mass descends from rest under the action of gravity, causing the wheel to turn as it descends; prove that when m' has descended h feet from rest its velocity is

$$\sqrt{\frac{2m'gh}{m+m'}} \text{ velos.}$$

13. Two masses m lbs., m' lbs. are fastened to the ends of a light string, one of them m is placed on the slant side of a smooth inclined plane of inclination a, and the other m' hangs freely suspended under the action of gravity by the string which passes over the topmost point of the plane; the masses are moving with uniform velocity; prove that

$$m' = m \sin a.$$

14. A rifle ball moving with 2000 velos has its velocity reduced by 200 velos in passing through a plank; how many such planks would it penetrate, assuming the same amount of work to be performed in overcoming the resistance of each plank.

15. Prove that a foot poundal is 421,394 ergs.

16. A particle, in the form of a ring, slides on a smooth fixed curved wire under the action of gravity; shew that the velocity of the ring at any instant is equal to that due to a fall from a certain horizontal plane to its position at that instant.

17. Assuming that the energy of a projectile is constant, prove that its velocity at any instant is equal to that due to a fall from a certain horizontal line to its position at that instant.

18. A simple pendulum of length l feet just describes a complete revolution; prove that the tensions of the rod in its two vertical positions are in the ratio of 1 to 5.

19. Shew that when a simple pendulum just describes a complete revolution the tension of the rod vanishes at a certain instant; determine the position of the rod at that instant.

20. The velocity which a simple pendulum of length l feet must have at its lowest point that, when it reaches its highest point the rod may have no tension, is $\sqrt{(5lg)}$ velos.

21. A regular polygon is made of some rigid material; $A, B, C, D\ldots$ are the middle points of the sides; a smooth perfectly elastic particle under the action of no forces moves along AB with velocity v velos, and impinges at B on that side of the polygon whose middle point is B.

Prove that the particle will proceed to describe the polygon A, B, C, $D\ldots$ with uniform speed v velos.

Now suppose the polygon to have an infinite number of sides, the length of each side being infinitely small; then it follows that the polygon A, B, C, $D\ldots$ coincides with the material polygon; and we have an independent proof of Art. 180.

CHAPTER XVI.

POWER.

183. We use the word '**agent**' to denote a *machine* which transforms some stored up chemical or other energy into work.

Examples. A *navvy* transforms the chemical energy of his food into work when he raises an embankment.

A *steam-engine* transforms the energy stored in coal into work when it does work on a train.

A *windmill* transforms the kinetic energy of the air into work when it pumps up water, etc.

184. DEF. The **power** of an agent varies as the amount of work which the agent can do in a unit of time.

The **unit** *power* is the power of an agent which can do 1 foot-poundal per second.

Hence the **measure** of the power of an agent is the *number* of foot-poundals it can do per second.

185. A **Horse-power** is the *power* of an agent which can do 550 foot-pounds per second;

that is, $550 \times g$ foot-poundals per second.

Example i. *At what uniform speed can a* horse *of* 1 horse-power *draw a tram-car of* 1 ton, *supposing the friction etc. to cause a uniform resistance of* 50 lbs. weight?

Suppose the required speed to be v velos.

Then the tram passes over v feet per second.

Therefore the horse does $v \times 50$ foot pounds per second.

He can do 550 foot pounds per second.

Hence $v = 11$.

That is, he can draw the car at the rate of 11 velos;
or, at the rate of 7½ miles per hour.

Example ii. *At what uniform* speed *can a man of* $\frac{1}{10}$ horse-power, *draw* 1 ton *up a smooth inclined plane of* 1 in 10?

The sine of the angle of inclination is $\frac{1}{10}$.

Therefore the force necessary to move the ton up the plane is $\frac{1}{10}$ of a ton weight; or, 224 lbs. weight.

[Now it is very unlikely that the man can exert such a force, unaided.

To do so, he must make use of a wheel and axle, or some such mechanical contrivance, which by multiplying the distance through which the muscular force of the man works, divides by the same number the force which must be exerted.]

Let the velocity required be v velos,

then, $$v \times 224 = 55,$$

or, $$v = \frac{55}{224} = \frac{1}{4} \text{ (very nearly)}.$$

That is, he can (by the aid of a machine) draw the ton up the plane at the rate of about 15 feet per minute.

EXAMPLES. XLI.

1. What must be the power of an engine that can keep a train of 100 tons going at the rate of 60 miles an hour on horizontal rails when the friction is equivalent to a horizontal resistance of $\frac{1}{80}$ of the weight of the train?

2. At what rate could the engine of Question **1** draw the same train up an incline of 1 in 50, the resistance of friction etc. being unaltered?

3. Suppose the same train (Questions **1** and **2**) to be on a horizontal plane, what velocity could the engine attain from rest in 100 yards?

4. Supposing the same train to be descending an incline of 1 in 50, what velocity could the engine give the train from rest in 100 yds.?

5. A horse can draw a cart of 1 ton at the rate of 4 miles an hour; supposing that his power is 1 horse-power, find the resistance of friction.

6. What speed from rest could the horse in Question **5** give to the cart in 100 yds.?

7. What must be the power of an engine which can raise 1000 gallons of water per minute a height of 20 feet, delivering it by means of a uniform pipe of which a linear foot contains 1 gallon?

8. A man throws a brick of 7 lbs. a height of 8 feet once in a second; at what rate is he working?

CHAPTER XVII.

ENERGY IS INDESTRUCTIBLE.

186. When a machine does work it receives a supply of some kind of energy which it transforms into mechanical work.

A man who raises weights receives a supply of energy by the chemical combustion of the food which he eats.

187. A machine which has no energy supplied to it cannot do work; it can only *transmit* work.

A system of pulleys used in raising weights transmits the work given to it by what is termed, by many writers on Statics, '*the power*'; this work is transmitted to what is called '*the weight.*'

It will be found that (when the kinetic energy is unchanged and friction is neglected) the work done by '*the power*' is always equal to the work done on '*the weight.*'

For example, if the weight is 16 times the power, the power will have to pass over a distance of 16 feet in raising the weight 1 foot.

188. No machine however can be made in which the resistance of *friction* is entirely excluded; so that no machine can transmit the *whole* of the work put into it.

This friction, which absorbs work, uses the work it absorbs in heating the parts of the machine; it changes some of the work put into the machine into another form of energy which we call *heat*.

189. Energy is indestructible; therefore energy cannot be produced out of nothing; for that which can be produced can be destroyed.

One form of energy can be obtained from another; but it can be obtained in no other way.

A steam engine does work; it is supplied with the energy of heat obtained from the combustion of coal; in which coal is stored the energy radiated from the sun countless years ago.

190. Mechanical work involves the moving of the surface of one mass over that of another. This motion invariably sets up friction which absorbs energy, converting it into heat. Heat tends to diffuse itself in all directions. And since heat can only be converted into mechanical energy when the heat is highly *concentrated*, it follows that the energy absorbed by friction is dissipated in such a way that it is lost for all practical purposes.

191. Hence, notwithstanding that energy is indestructible, yet kinetic energy is continually being eaten away, as it were, by friction and converted into the unserviceable form of uniformly diffused heat.

For example, it is proved in Art. 133 that the kinetic energy of two masses is diminished by their mutual impact. The reason of this loss of kinetic energy is that during an impact the particles of the masses in the neighbourhood of the point of impact are first compressed and then recover more or less their original position. This compression and recovery generates *heat*; accordingly the kinetic energy of the two masses is diminished by the amount of energy taken up by the heat generated.

192. Heat is itself the kinetic energy of the minute particles of a substance.

Gas consists of minute invisible particles of matter; these particles are in motion; they impinge repeatedly and ceaselessly on one another. They are perfectly elastic (possibly because they are too small to be le of compression); consequently by Art. 133 the joint energy of rticles is unaltered by their impact.

The quantity of heat contained in a certain amount of gas is *the kinetic energy* of all its particles. This energy (if none be allowed to pass away from the gas) continues unaltered because the particles of the gas are perfectly elastic.

193. The main principle which it is the object of Dynamics to explain and establish may be summed up in the words

Energy is indestructible.

'The discussion of the various forms of energy—gravi'tational, electro-magnetic, molecular, thermal &c.—with 'the conditions of the transference of energy from one form 'to another, and the constant dissipation of the energy 'available for producing work, constitutes the whole of 'physical science,—in so far as it has been developed in 'the dynamical form,—under the various designations of 'Astronomy, Electricity, Magnetism, Optics, Theory of the 'Physical states of bodies, Thermo-dynamics and Chemistry.' *Professor Maxwell's Matter and Motion*, p. 95.

EXAMINATION PAPERS.

I. CAMBRIDGE PREVIOUS EXAMINATION. PART III. [ADDITIONAL SUBJECTS.] *October*, 1886.

(Questions 1 and 2 Trigonometry.)

3. Explain what is meant by uniform acceleration.

A stone is thrown vertically upwards with a velocity of 36 feet per second. After what times will its velocity be 12 feet per second?

4. A particle starts from rest and moves for t seconds with an acceleration f feet per second per second. Prove that the space described is $\frac{1}{2}ft^2$ feet.

Find the space described by a falling stone in the eleventh second of its fall.

5. A railway train moving with velocity 48 miles an hour has its velocity reduced to 16 miles an hour in five minutes. Find the space passed over in the interval, the retardation being assumed to be uniform.

6. Enunciate and illustrate Newton's first and second laws of motion.

A certain force acting on a particle of mass M produces an acceleration f. Shew that, if suitable units be chosen, Mf may be taken as the measure of the force.

7. A bullet is projected at an inclination θ to the horizon ($\cos\theta = \frac{4}{5}$) with a velocity of 1200 feet per second. Find the greatest height it attains and its range on a horizontal plane through the starting point.

8. Find the time that a particle takes to slide from rest down a smooth plane of length a which is inclined at an angle α to the horizon.

Shew that the times of descent of a particle down all chords drawn through the highest point of a vertical circle are equal.

II. CAMBRIDGE PREVIOUS EXAMINATION. PART III. [ADDITIONAL SUBJECTS.] *October*, 1886.

(Questions **1** and **2** Trigonometry.)

3. Explain what is meant by saying that the acceleration of a falling stone is g feet per second per second.

A stone is thrown vertically upwards with a velocity of 48 feet per second. After what times will its velocity be 16 feet per second?

4. Shew that a particle which moves from rest with an acceleration a describes a space $\frac{1}{2}at^2$ in t units of time.

A stone takes 12 seconds to fall to the ground. Find the space described in the last second.

5. A railway train moving with a velocity of 32 miles an hour has its velocity reduced to 8 miles an hour in 5 minutes. Find the space passed over in the interval, the retardation being assumed to be uniform.

6. Enunciate and illustrate Newton's second and third laws of motion.

A force F acting on a certain mass produces an acceleration a. Shew that, if suitable units be chosen, $\frac{F}{a}$ may be taken as the measure of the mass.

7. A bullet is projected with a velocity 1400 feet per second at an inclination θ to the horizon ($\cos\theta = \frac{4}{5}$). Find the greatest height it attains and its range on a horizontal plane through the starting point.

8. Find the space described and the velocity acquired in t seconds by a particle sliding from rest down a smooth plane inclined at an angle a to the horizon.

Shew that the times of descent of a particle down all chords drawn through the lowest point of a vertical circle are equal.

III. CAMBRIDGE PREVIOUS EXAMINATION. PART III. [ADDITIONAL SUBJECTS.] *December* 11, 1886.

Question **1.** Trigonometry.

2. How is acceleration measured when variable?

Prove the formula for uniform acceleration $v^2 = 2fs$.

3. Enunciate and prove the Parallelogram of Velocities.

A stone is dropped from a height of 4 feet above the floor of a railway carriage by a person in the carriage. Find its velocity on striking the floor if the train is travelling at 15 miles an hour.

4. A ball is thrown vertically upwards with velocity g, and one second later another ball is thrown up from the same point with velocity $2g$. When and where will it strike the first ball?

5. Shew from the second law of motion that if a body be acted upon by a given force the acceleration produced varies inversely as the mass of the body.

The forces acting upon a train have a resultant in the direction of motion equal to the weight of 1155 lbs. Find the mass of the train if it acquires a velocity of 60 miles an hour from rest in 8 minutes.

6. Find the time of sliding from rest down a length a of a smooth plane inclined at an angle α to the horizon.

If the times of sliding from rest at A down two straight lines AB, AC are proportional to their lengths, shew that BC is horizontal.

7. Masses M and m hang at the ends of a string which passes over a smooth pulley. Find the acceleration.

If M strike the ground one second after the commencement of the motion, how much longer will it be before m first comes to rest?

8. Prove that the greatest height attained by a projectile above the horizontal plane through the point of projection is equal to $\frac{v^2}{2g}$, where v is the vertical component of the velocity of projection.

If the greatest height is equal to the range, find the tangent of the angle of projection.

IV. CAMBRIDGE PREVIOUS EXAMINATION. PART III. [ADDITIONAL SUBJECTS.] *December* 11, 1886.

Question **1.** Trigonometry.

2. How is velocity measured when variable?

Prove the formula for uniform acceleration $2s = vt$.

3. Shew how to find the relative velocity of two particles whose velocities are given in magnitude and direction.

Shew that if two particles are projected in any manner under the action of gravity their relative velocity is constant throughout the motion.

4. A ball is thrown vertically upwards with velocity g, and one second later another ball is thrown up from the same point with velocity $3g$. When and where will it strike the first ball?

5. Shew from the second law of motion that the acceleration produced by a force acting upon a given mass is proportional to the force.

The forces acting upon a train whose mass is 100 tons have a resultant in the direction of motion equal to the weight of 1000 lbs. In what time will it acquire from rest a velocity of 60 miles an hour?

6. Find the velocity acquired in sliding from rest down a length a of a smooth plane inclined at an angle α to the horizon.

If BC be horizontal, prove that the times of sliding down any two straight lines AB, AC from rest at A are proportional to their lengths.

7. Masses M and m hang at the ends of a string which passes over a smooth pulley. Find the tension of the string when motion takes place.

If M strike the ground one second after the commencement of the motion, find the distance of m from the ground when it first comes to rest, M and m being originally at the same level.

8. Prove that the range of a projectile on a horizontal plane is equal to $\frac{2uv}{g}$, where u, v are the horizontal and vertical components of the velocity of projection.

If the greatest height attained be 36 feet, prove that v is equal to 48.

V. FIRST M.B. EXAMINATION. CAMBRIDGE, *June*, 1886.

1. Define the terms *velocity*, *momentum*, *acceleration*, *force*, and state the relation between the force producing motion in a body and the motion produced.

A body falling freely acquires at the end of one second a velocity of 32 feet per second. Masses of 1 and 3 lbs. hang from the two ends of a fine string suspended over a smooth pulley. At what rate will they be moving at the end of one second after they are set free?

2. Two heavy bodies are dropped at the same time, one from a height of 25 and the other from a height of 50 feet, find the height and velocity of the second when the first touches the ground.

3. What do you understand by work and energy? How much energy has a mass weighing 1 cwt. and moving at the rate of 100 yards per second?

In what units is your answer expressed?

4. Explain carefully how to find the resultant of two forces acting at a point.

A body weighing 4 lb. at rest on a smooth table is acted upon by forces of 3 and 4 lb. weight in directions oblique to the table and at right angles to each other. Shew by a diagram how to find the directions of the forces.

5. What do you understand by the mechanic advantage of a machine?

In a certain system of pulleys it is found that the power descends 1 foot, while the weight rises 1 inch. What power will be required to raise a weight of 1 cwt.?

VI. OXFORD AND CAMBRIDGE SCHOOL EXAMINATION FOR HIGHER CERTIFICATES, 1879.

7. Define uniformly accelerated motion.

Shew what must be the ratio of the height to the length of a smooth inclined plane in order that a body may be four times as long in sliding down the plane as it would be in falling freely down the height of the plane, starting from rest in each case.

8. Prove the formula, $v^2 = u^2 + 2fs$.

A body after moving from rest a distance s down a smooth plane of inclination α comes to a plane at a less inclination β: find the impulse which must be given to it in order that it may begin to move down the second plane with the same velocity as it had when it came to the bottom of the first plane.

Prove also that as great a force will be required to stop it when it reaches the bottom of the second plane as if it had fallen freely from its starting point through the same height.

9. Two men, of weights w_1, w_2 respectively, start from the same point and walk at uniform rates in directions at right angles to each other: shew that the motion of their centre of gravity is uniform. If the rates at which the men walk are inversely as the square roots of their weights, prove that the rate of motion of their centre of gravity will bear to the rate of the first walker the ratio $\sqrt{\dfrac{w_1}{w_1 + w_2}}$.

10. Prove that the velocity at any point in the path of a projectile is that due to falling from the directrix to that point.

A perfectly elastic ball is dropped from the top of a tower of height h, and when it has fallen half-way to the ground it strikes a rigid projecting stone inclined at 45^0 to the horizon: determine where it will reach the ground.

11. Given unequal weights w_1, w_2 hang by the same string over a smooth pully: find the tension of the string at any time during their motion.

If after acquiring a certain velocity v the lighter weight catches up another weight sufficient to make it equal in weight to the heavier, determine the tension of the string at the instant, and the loss of vis viva, supposing the whole inelastic.

VII. OXFORD AND CAMBRIDGE SCHOOLS EXAMINATION FOR HIGHER CERTIFICATES 1885.

7. State and prove the parallelogram of velocities.

From the window of a train which is moving at the rate of 60 miles an hour, a stone, thrown at right angles to the train with a velocity of 66 feet a second, hits a tree which is 90 ft. from the train at the time it is struck. How far was the window from the tree when the stone was thrown?

8. Define acceleration; explain what is meant by inertia.

'The number of units of weight in any particle is equal to $32 \cdot 2 \times$ the number of the units of mass in the same particle.' Under what circumstances is this statement true?

Find the relation between the measure of the weight and of the mass of a particle when a mile and a minute are the units of length and of time respectively.

9. Prove that the velocity of a projectile in vacuo under the action of gravity at any point in its path is equal to that which a particle would have if it fell vertically from the directrix of the parabola to that point.

10. Two weights W_1, W_2 are connected by a string; one of them W_1 lies on a smooth inclined plane, angle a, the other W_2 hangs over the lower edge of the plane suspended by the string under the action of gravity; the two weights are in motion in a vertical plane. Find the tension of the string.

Find how far W_1 will move from rest in two seconds, when $W_1 = 12$ lbs., $W_2 = 4$ lbs., $a = 30^0$.

11. Define an impulse.

Explain what is meant by the coefficient of restitution in impact.

A gun of 40 tons projects a bolt of 2 cwt. Find how far the recoil of the gun will force it up a smooth inclined plane whose angle is 30°, if the initial velocity of the bolt when fired is 2000 ft. per second.

VIII. UNIVERSITY OF LONDON. B.A. AND B.SC. PASS EXAMINATIONS: 1886.

4. Shew how two coexistent velocities or accelerations may be compounded by the triangle of velocities and accelerations: and give illustrations.

Knowing the direction of the true wind, and the velocity and direction of the apparent wind on the ship as shewn by the vane on the mast, determine the velocity of the ship, supposing there is no lee-way.

5. Determine what would have to be the numerical value of g, the acceleration of gravity, in order that a body starting from rest should fall 10,000 feet in 10 seconds.

When $g=32$, determine—

(i) The distance fallen from rest in 10 seconds;

(ii) The time of falling 10,000 feet;

(iii) The initial vertical velocity in order that the body should fall 10,000 feet in 10 seconds.

6. Prove that a train going 45 miles an hour will be brought to rest in about 378 yards by the brakes, supposing them to press with two-thirds of the weight on the wheels of the engine and brake-vans, which are half the weight of the train; and supposing a coefficient of friction ·18.

7. Prove that an engine capable of exerting a uniform pull of 3 tons can take a train of 120 tons on the level from rest at one station to stop at the next station two miles off in about 3 minutes 38½ seconds, the speed being kept uniform when it has reached 45 miles an hour, and the brakes being applied as in the last question in order to stop.

(Neglect passive resistances, and take $g=32$.)

8. Prove that a piece of mud thrown from the top of a hansom cab wheel of diameter d feet, the cab moving with velocity v feet per second, will when it strikes the ground be at a distance $\frac{1}{4}v\sqrt{d}$ in front of the position then occupied by the point of contact of the wheel with the ground.

Prove that the velocity v must exceed $4\sqrt{d}$ for the piece of mud to clear the wheel.

9. Determine the horse-power transmitted by a belt moving with a velocity of 600 feet a minute, passing round two pulleys, supposing the difference of tension of the two parts is 1650 lbs.

10. A pile is driven a feet vertically into the ground by n blows of a steam-hammer fastened to the head of the pile.

Prove that, if p is the mean pressure of the steam in lbs. per sq. inch, d the diameter of the piston in inches, l the length of the stroke in feet, w the weight in lbs. of the moving parts of the hammer, and W the weight of the pile and the fixed parts of the steam-hammer attached to it; then the mean resistance of the ground in lbs. is

$$\frac{nw}{W+w}\left(w+\tfrac{1}{4}\pi d^2 p\right)\frac{l}{a}.$$

IX. Examinations for Scholarships at St John's College.

December, 1885.

9. Enunciate and prove the parallelogram of velocities.

A number of soldiers are marching with shouldered rifles in a shower of rain which falls vertically. Having given the velocity of the rain-drops and the rate of marching, find what must be the least inclination of the rifles to the vertical in order that no rain may enter the barrels.

10. Define acceleration, and explain how it is measured.

Taking 32 as the measure of the acceleration of a body falling under the action of gravity when a foot and a second are units of length and time, find its measure when the units are a mile and two minutes forty-five seconds.

If the acceleration of a falling body be taken as the unit of acceleration, and if the unit of velocity be the velocity it acquires in four seconds, find the units of length and time.

13. A ball moving with velocity v impinges directly on a ball at rest; this ball moves with the velocity communicated, and after a direct rebound from a wall comes again into collision with the first ball. Find the ratio of the masses if the first ball be reduced to rest.

15. Define the terms *potential energy of a system* and *kinetic energy of a system*, and find an expression for the potential energy of an elastic string when extended beyond its natural length.

Two equal particles on a smooth horizontal plane are connected by an elastic string. If the particles be drawn apart until the length of the string is double its natural length and then let go, find the greatest velocity acquired by the particles.

16. Find the direction and magnitude of the acceleration of a point which describes with given uniform velocity a circle of given radius.

If the particles in question **15**, when drawn apart to a given distance, be projected with equal velocities in opposite directions at right angles to the string, find the velocity of projection in order that the particles may each move uniformly in the same circle. Prove that the velocities requisite for circular motion when the lengths of the string are respectively double and quadruple its natural length are in the ratio of $1 : \sqrt{6}$.

X. ENTRANCE SCHOLARSHIP EXAMINATION. GONVILLE AND CAIUS COLLEGE. *December*, 1885.

8. Define the acceleration of a moving point and state how many quantities are necessary to specify it.

The Moon's distance from the Earth is about 239,000 miles and she revolves once round the Earth in about $27\frac{1}{3}$ days. Find her acceleration relatively to the Earth with feet and seconds as units.

9. Shew that Newton's Second Law provides a direct method of comparing the magnitudes of two forces. A wedge mass M and angle α is placed on a smooth horizontal plane, and a particle mass m is placed on the wedge. Shew that the acceleration of the particle with respect to the wedge is

$$g \sin \alpha \, \frac{M+m}{M+m\sin^2 \alpha}.$$

10. A particle is projected with given velocity under the action of gravity from a point P so as to pass through a point Q; shew that if t_1, t_2 are the two possible times of flight

$$gt_1t_2 = 2PQ.$$

11. State the experimental law on which the determination of the motion of two elastic balls after impact depends, and shew that the Kinetic Energy after impact is never greater than that before.

Three particles of equal mass are placed at the angular points A, B, C of an equilateral triangle and A, B are connected with C by equal strings which are just taut. C is projected perpendicularly towards AB. Find the loss of kinetic energy after A and B are jerked into motion.

12. When does a force do work on a body, and how is the work done measured?

Apply the methods of energy and conservation of angular momentum to the following problem. A particle which can move on a smooth horizontal table is attached to a fixed point of the table by an elastic unstretched string of length a, which is such that the weight of the particle would stretch it to twice its natural length. Prove that if the particle be projected with velocity u perpendicular to the string, its velocity when next moving at right angles to the string will be given by

$$v^3 + v^2u + vga - uga = 0.$$

XI. ENTRANCE SCHOLARSHIP EXAMINATION. GONVILLE AND CAIUS COLLEGE. *December*, 1886.

5. State and exemplify the proposition known as the "parallelogram of velocities."

AB, CD are two lines, bisecting each other at right angles. Points start at the same instant from A, C and move along AB, CD with uniform velocities. Shew that, the relative velocity of the points being given, the time that elapses before they are at their shortest distance apart is proportional to the sum of their speeds.

6. State the Second Law of Motion; and thence shew that a number of forces acting on a material particle may be represented by straight lines and follow the same law of composition as velocities.

OA, OB, OC, etc.... represent an equilibrating system of forces acting at a point in a plane, and a transversal meets them in a, b, c, etc....: shew that

$$\frac{OA}{Oa} + \frac{OB}{Ob} + \frac{OC}{Oc} + \text{etc....} = 0.$$

9. Calculate the velocity of the moving weights in Atwood's machine in terms of the space described from rest.

Explain generally how the inertia of the moving wheel will affect the result: and calculate its effect supposing its mass wholly concentrated in its rim.

10. State the experimental result used in calculating the result of impact between smooth spheres.

Shew that the energy interchanged between the particles, if perfectly elastic, is $\frac{2PQ}{(P+Q)^2}(u \sim v)(Pu+Qv)$ where P, Q are their masses, and u, v their velocities along the line of impact.

11. Explain what is meant by (i) work, (ii) kinetic energy; and illustrate your explanation by considering the flight of a projectile.

A shot weighing 500 lbs. fired from a gun, whose barrel is 10 inches in diameter and 22 feet long beyond the powder chamber, receives a muzzle velocity of 2100 feet per second. Assuming that the energy expended in expelling powder gases, producing rotation of the shot, etc. is one third of the energy of translation of the shot on leaving the gun, find the mean pressure per square inch in the barrel during discharge.

XII. COLLEGE EXAMINATION.

CLARE COLLEGE, GONVILLE AND CAIUS COLLEGE, KING'S COLLEGE.

June, 1884.

1. State Newton's laws of motion and deduce the relation between the acceleration of a mass and the force acting on it.

Two masses m, m' are attached by an inelastic string, m' rests on an inclined plane of mass M and m hangs vertically over the upper end of the plane pressing against the vertical face of the plane. If the inclined plane be free to slide parallel to a line of greatest slope along a horizontal line, find its acceleration and the pressures between it and the particles.

2. Define the following quantities, explaining how they are measured and mentioning their dimensions; Work, Power, Kinetic Energy, Potential Energy, Impulse. Justify the measure you give for Kinetic Energy.

grav cannon ball weighing 10,000 grammes, is discharged with a
if t_1, t_2 al 45,000 centimetres per second from a cannon, the length of
l is 200 centimetres, prove that the mean force exerted on
11. State the explosion is $5 \cdot 0625 \times 10^{10}$ dynes.
motion of tw icle is projected with a given velocity in a direction
Kinetic Energy angle with a line of greatest slope of an inclined plane

and in the same vertical plane with it; find the range on the plane and the greatest distance from the plane which the particle attains.

A particle, whose coefficient of restitution is e, is projected vertically with velocity u from a point O on a plane inclined at an angle α to the horizon; prove that the distance from O of the point where the particle meets the plane for the $(n+1)$th time is

$$\frac{2u^2}{g}\sin\alpha\,\frac{e(1-e^{n-1})(1-e^n)}{(1-e)^2},$$

and prove that the points of the different parabolic paths where their tangents are parallel to the inclined plane lie on a parabola.

4. Shew how to determine the velocity and acceleration of the centre of inertia of any given masses, knowing the velocities and accelerations of each mass; and prove that the kinetic energy of two masses is equal to the kinetic energy of the two supposed moving with the velocity of the centre of inertia, and the kinetic energy of each mass relatively to the centre of inertia.

Three masses each equal to m are attached to the extremities and middle of a rod without inertia, and of length $2a$. A blow B is applied perpendicularly to the rod at a distance x from the middle point, prove that the kinetic energy generated is

$$\frac{B^2}{6m}\left\{1+\frac{3}{2}\frac{x^2}{a^2}\right\}.$$

5. A particle of mass m revolves in a circle of radius r with angular velocity ω, prove that the force is necessarily to the centre and find its value.

Two particles of masses m, m' lie on a smooth horizontal table connected by an inelastic string of length a, m is projected in the plane at right angles to the string, prove that the initial radius of curvature of its path is $\frac{m+m'}{m'}a$.

XIII. COLLEGE EXAMINATION.

CLARE COLLEGE, GONVILLE AND CAIUS COLLEGE, KING'S COLLEGE.

June, 1885.

1. Give Newton's three laws of motion and shew from the second law that the measure of a uniform force is the momentum added on per unit of time.

A particle (mass m) slides down a smooth wedge (mass M, inclination α) which is free to move on a smooth horizontal plane. Prove that the path of the particle is a straight line, and assuming the principle of energy prove that its vertical acceleration is

$$\frac{g(M+m)\tan^2\alpha}{M+(M+m)\tan^2\alpha}.$$

2. Prove the formula $s = ut + \frac{1}{2}ft^2$ for rectilinear motion with uniform acceleration.

A cylinder of height h and diameter d is standing on the horizontal seat of a railway carriage. If the train begins to move with acceleration f, prove that the cylinder will not remain undisturbed unless f is less than each of the quantities μg and $\frac{dg}{h}$ where μ is the coefficient of friction.

3. Prove that the time of sliding down all chords of a vertical circle to the lowest point is the same.

Find the position of a point on the circumference of a vertical circle, such that the time of descent from it down the radius to the centre and down a chord to the lowest point may be the same.

4. Explain the theory of Atwood's machine.

Two weights P and W balance on the wheel and axle. Prove that if they be interchanged the weight W will after one second be descending with a velocity

$$g\frac{W(W-P)}{W^2-WP+P^2},$$

the mass of the axle being neglected.

5. Prove that the path of a projectile is a parabola; and that the velocity at any point is that which would be acquired by a particle falling from the directrix to that point.

A perfectly elastic particle is let fall from a point P on the upper surface of a vertical hoop (centre O) and rebounds from the inner surface of the hoop. Prove that after two rebounds it will be moving vertically if

$$\tan\theta = 2\sin 4\theta,$$

where θ is the angle which OP makes with the vertical, and find the time which elapses before the particle arrives again at P.

6. A particle is describing a circle of radius r with velocity v, shew that the acceleration along the radius is $\frac{v^2}{r}$.

A string of length l has its ends fastened to two points A, B in the same vertical line, and a bead P on the string rotates uniformly about AB so that BP is always horizontal; prove that its angular velocity is equal to $l\left(\frac{2g}{a(l^2-a^2)}\right)^{\frac{1}{2}}$, where $AB = a$.

XIV. College Examination.

Clare College, Gonville and Caius College, King's College.

June, 1886.

1. Give the law of motion connecting mass, force and acceleration for a moving particle. Do you consider it to be an experimental result, and if so in what respects, or definition, and if so of what?

What are the ultimate tests of equality (1) of two forces, (2) of two masses?

2. What choices as to units are commonly made in dynamics before any equations are written down?

If, other choices of units being the same as usual, we choose for unit of velocity the velocity which a particle of unit mass acquires in moving from rest through a unit length under the action of unit force, find the velocity of a body which moves through ten units of length per unit time.

3. Find the acceleration of a particle which is moving with uniform velocity in a circle.

A railway carriage is moving in a horizontal plane at the rate of 30 miles an hour round a circular curve of which the radius is 10 chains. In the centre of the carriage, and fixed with regard to it in a horizontal plane, is a smooth circular wire of 6 ft. diameter, on which a bead moves uniformly, making 20 revolutions per minute; the two circular motions being in the same direction. Find the pressure between the bead and the wire.

4. Give the arrangement and theory of Atwood's machine.

In a certain experiment with this machine, the two weights which are supposed to be equal differ by a small weight w, while their sum is correctly known, and the given distances moved through in the two stages of the motion are equal. Shew that error, due to w, in the value found for the acceleration of gravity is approximately $\frac{w}{2}\left(\frac{3}{m}+\frac{1}{M}\right)$;

where m is the mass removable, and M is the whole moving mass (including the allowance for the inertia of the pulley) after the removal of m.

5. Two inelastic particles are moving in one straight line; deduce from first principles the effect of a collision between them.

A goods train consists of a number of similar wagons and an engine whose mass is an integral number, μ, times that of a wagon. They are coupled together by chains of equal lengths, inelastic and without weight. The train is initially at rest, on a straight line of railway, with all the vehicles in contact and the couplings slack. The engine then begins to move, the steam exerting a constant tractive force, and each wagon is started with a jerk as its coupling tightens. Shew that, during the starting, the velocity of the moving part of the train will be greatest just before the n^{th} impact, where $n = 2\mu^2 - 3\mu + 1$, provided that there are at least n wagons. All rotation of wheels and friction may be neglected.

6. Find the path of a projectile in a vacuum.

A solid paraboloid of revolution is fixed with its axis vertical and vertex uppermost; a particle is projected from the vertex with a given velocity, and impinges on the surface after a parabolic flight. Find the angle of projection for which the time of flight is greatest.

XV. MATHEMATICAL TRIPOS, 1885.

1. Distinguish between the weight of a body and its mass. How does the weight of a body vary in different latitudes, and how would it be affected if the body were removed to the surface of the moon? Shew how the intensity of gravity at the surface of a planet fixes a superior limit to the dimensions to which its inhabitants can attain.

7. Find the range on a horizontal plane of a shot for a given angle of elevation and initial velocity; and shew that the height of the vertex in feet is approximately four times the square of the time of flight in seconds.

Prove that, projected on a vertical target, the shot, as seen from the point of projection, will appear to descend with constant velocity.

8. Find the acceleration of a body sliding up or down a rough inclined plane; and prove that the velocity at any point will be that due to falling freely under gravity from a certain straight line which slopes downwards in the direction of motion at the angle of friction.

Prove that the loss of time in going from A to C, two points on a railway at the same level 8 miles apart, due to an incline of 1 in 100 from A up to B, and an incline of 1 in 300 from B down to C, instead of going on a level line from A to C at a uniform velocity of 45 miles an hour, is about 2 min. 20 sec.

It is supposed that with full steam on the velocity drops from 45 miles an hour at A to 15 at the summit B, and that in descending the incline from B to C full steam is still kept on till the velocity has again reached 45 miles an hour, after which the velocity is kept uniform by partly shutting off steam; and prove that this happens at a point Q distant from B about 1 mile 892 yards.

9. Prove that a train of W tons going up an incline of 1 in m will acquire velocity $\left(\frac{P}{W}-\frac{1}{m}-\frac{\mu}{2240}\right)gt$, and energy $\frac{1}{2}W\left(\frac{P}{W}-\frac{1}{m}-\frac{\mu}{2240}\right)^2 gt^2$ foot-tons, in $\frac{1}{2}\left(\frac{P}{W}-\frac{1}{m}-\frac{\mu}{2240}\right)gt^2$ feet, after t seconds from rest, if P denotes the pull of the engine in tons, and μ the resistances in pounds per ton.

Prove that in Question 8 the pull of the engine from A to Q is $2\frac{5}{48}$ tons, and from Q to C is $\frac{7}{12}$ of a ton, supposing $W=200$, $\mu=14$, $g=32$; and find the extra expenditure of work due to the inclines.

10. If a body attached to its centre of mass to one end of a string of length r, the other end of which is attached to a fixed point on a smooth horizontal plane, makes n revolutions a second, the tension of the string is to the pressure on the plane as $4\pi^2n^2r$ to g.

Prove that, if a train is running round a curve of radius r with velocity v, the weight of a carriage is divided between the outer and inner rail in the ratio of $gra+v^2h$ to $gra-v^2h$, where h is the height of the centre of gravity of the carriage above the rails, and $2a$ the distance between them.

11. Calculate the velocities after direct impact of two elastic spheres.

An inelastic pile of w lb. is driven vertically a feet into the ground by n blows of a hammer of W lb., falling h feet. Prove that $\frac{nW^2}{W+w}\frac{h}{a}$ lb. superposed on the pile in addition to W would drive it down very slowly, supposing the resistance uniform.

If the pile is crushed x feet by each blow, where x is small, the mean pressure exerted by the hammer is $\frac{Ww}{W+w}\frac{h}{x}$ lb., and each blow lasts for $\frac{x}{h}$ of the time of falling of the hammer, neglecting forces not due to the impulse.

XVI. MATHEMATICAL TRIPOS, 1886.

7. Define momentum, and explain how the momentum of a particle is altered by the action of a given force.

A smooth particle of weight W is free to move in a fine tube which is inclined to the horizon at angle α and is constrained to move upwards with a constant vertical velocity V; prove that the pressure on the tube is $W\cos\alpha$. Prove also that if the tube be initially at rest and the particle attached to it by a very fine thread, and if the velocity V with which the tube is started off be at least sufficient to break the thread, then at a time t after starting the possible positions of the particle in the tube cannot differ by a distance exceeding $Vt\sin\alpha$.

8. Prove the formula $s=vt+\frac{1}{2}ft^2$, and investigate the relative path of two beads free to move on two straight smooth wires in the same plane under the action of constant forces.

A telescope on a heavy platform is drawn up a smooth inclined plane of inclination α by a force so adjusted that the telescope may keep a given projectile always in the field of view. Prove that if β be the angle the telescope makes with the plane and V the initial velocity of the projectile perpendicular to the axis of the telescope, the magnitude of the force per unit mass must be $g\cos\alpha\cot\beta$ and the initial velocity of the telescope $V\operatorname{cosec}\beta$. The motion of all the bodies is supposed to be in one plane.

9. Prove that when a heavy particle is to be projected from a given point with a given velocity to pass through another given point there are two possible directions of projection.

A man standing on the edge of a cliff throws a stone with given velocity u at a given inclination in a plane perpendicular to the edge. After an interval τ he throws from the same spot another stone with given velocity v at an angle $\frac{1}{2}\pi+\theta$ with the line of discharge of the first stone and in the same plane. Find τ so that the stones may strike one another, and prove that the maximum value of τ for different values of θ is $2v^2/gw$, and occurs when $\theta=\sin^{-1}v/u$, w being v's vertical component.

10. Two elastic balls moving with given velocities strike one another directly. Find in terms of Newton's coefficient their velocities after impact and the loss of kinetic energy in the impact.

Two perfectly elastic balls of different masses are set in motion with velocities V, v within a smooth circular tube fixed in a horizontal position. If after the first impact they describe angles A_1, a_1 respectively before the second impact, and if after the second impact they describe angles A_2, a_2 respectively before the third impact, then

$$V\frac{A_1-A_2}{A_2}+v\frac{a_1-a_2}{a_2}=0.$$

11. Find the component, in any specified direction, of the acceleration of a particle moving with uniform velocity in a circle.

AB is a fine elastic string passing round a smooth vertical peg at a centre of force attracting with a force proportional to the distance. *AP* is a fine inextensible string carrying a series of masses, and *BQ* is another such string carrying another series. All the masses rest on a smooth horizontal plane and are set revolving about the centre with the same angular velocity, after being drawn out sufficiently far for them to continue in steady revolution with all the masses of each system in one straight line through the centre. Prove that for any two different intensities of the centre, provided the angular velocity remain the same, the difference between the total kinetic energies is proportional to the difference between the products of the momenta of the two sets of masses.

12. A heavy particle resting on the top of a smooth fixed sphere is slightly displaced. Find its pressure on the sphere at any point of its path.

Two smooth equal spheres are supported by fine equal strings attached to their surfaces and hanging from one fixed point. The spheres are now glued together at their point of contact, and a smooth cylinder is laid on them so as to rest with its axis horizontal and not to interfere with the strings. Supposing the glue suddenly to give way, find the ratio of the new instantaneous pressure of the cylinder on a sphere to the previous pressure, and prove that if each string make an angle 60° with the horizon and an angle 30° with the perpendicular from the centre of its sphere upon the axis of the cylinder, there is no change in the pressure, provided the masses of the spheres and cylinder be all equal.

ANSWERS.

I.

1. $27\frac{3}{11}$ velos. **2.** 12 min. $34\frac{2}{7}$ sec.

3. (i) $27\frac{3}{11}$ miles per hour. (ii) $3\frac{9}{22}$ miles per hour.

4. (i) $\frac{11}{6300}$ velos. (ii) $\frac{11}{90720}$ velos. **5.** $1536\frac{32}{63}$ velos.

6. 1 min. $1\frac{17}{43}$ sec. **7.** 500 yards in $11\frac{1}{4}$ sec.; their ratio is 66 to 100.

8. 11 to 6.

9. (i) 30 velos; $20\frac{5}{11}$ miles per hour.
(ii) $26\frac{46}{49}$ velos; $18\frac{18}{49}$ miles per hour.
(iii) $20\frac{380}{421}$ velos; $14\frac{106}{421}$ miles per hour.
(iv) $17\frac{1113}{5111}$ velos; $11\frac{3779}{5111}$ miles per hour.

10. (1) $35\frac{1}{5}$ velos. | (i) 24 miles per hour.
(2) $29\frac{4581}{5911}$ velos. | (ii) $20\frac{1780}{5911}$ miles per hour.
(3) $32\frac{141}{360}$ velos. | (iii) $22\frac{15}{176}$ miles per hour.

II.

1. $\frac{22c}{15}$ velos. **2.** $\frac{15hv}{22}$ miles. **3.** $\frac{3m}{k}$ seconds.

4. $\frac{45hm}{22t}$ miles. **5.** $40\lambda k$ *yards*. **6.** $\frac{22mt}{15k}$ hrs. **7.** $\frac{22nt}{45k}$ yards.

III.

1. $10\frac{1}{2}$ velos. **2.** (i) 10 velos. (ii) 11 velos. **4.** 4 velos; $33\frac{1}{3}$ yds.

5. $\frac{5a}{2}$ velos. **6.** $9v$ velos.

IV.

1. 13 velos. **2.** 55 velos.

3. 255 velos, 210 velos, 165 velos, 120 velos. **4.** 65 velos.

5. 277 velos. **6.** 3 min. 18 sec. (after it begins to decrease.)

7. (i) 4 sec. (ii) 1 m. $10\frac{2}{3}$ sec. **8.** 40 sec. previously.

9. 4 sec.; 8 sec. **10.** 11 sec. **11.** 96 ft.; 48 ft.; 96 ft.

12. $3\frac{1}{2}$ sec. previously; 7 sec. previously.

VI.

1. (i) 102 feet. (iii) 12,000 feet or $2\frac{3}{11}$ miles.
(ii) 208 feet. (iv) 27,600 feet or $5\frac{5}{22}$ miles.

5. 10 seconds before the 5 sec. **6.** Yes. **7.** 320 velos.

8. 81 ft. **9.** $48\frac{3}{4}$ feet. **10.** 32 ft.; $2\frac{1}{2}$ secs. previously.

11. In 3 secs.; 54 ft.

VII.

1. (i) 80 velos. (iv) 6720 ft.
(ii) 176 velos. (v) in $33\frac{7}{8}$ sec.
(iii) 320 ft. (vi) half a sec. previously.

2. 20 celos. **4.** 640 feet. **5.** 4 celos.

6. (i) 32 celos. (ii) 144 feet.

7. (i) 15 velos. (iv) 3 to 1.
(ii) 6 sec.; 90 ft. (v) $(6-3\sqrt{2})$ sec. $=1\cdot75\ldots$ secs.
(iii) $87\frac{1}{2}$ ft.

8. (i) -32 celos. (iv) it starts with 144 velos.
(ii) in $\frac{1}{2}$ sec. (v) in q sec.
(iii) -180 ft.

9. $\dfrac{k-2h}{3t^2}$ celos. **10.** $\dfrac{k-h}{2t+4}$.

VIII.

1. 3 sec. **2.** (i) $10(\sqrt{2}-1)$ sec. $=4\cdot14$ sec. (ii) $56\frac{1}{4}$ ft.

3. 100 sec.; 180000 ft. **4.** 40 sec.; 25600 ft. **5.** 6 celos.

6. 2694 velos (nearly). **7.** $\frac{44}{45}$ celos. **8.** 3·3 celos.

9. 2640 ft. **10.** $\frac{121}{3375}$ celos.

11. It has $-\frac{5}{11}$ celos, therefore the required interval t secs. is one of the roots of the equation $100=19\frac{6}{11}t-\frac{1}{2}\times\frac{5}{11}t^2$, that is $t=43\pm37\cdot5\ldots$

12. It has $-\frac{5}{48}$ celos; t is one of the roots of $400=13\frac{23}{24}t-\frac{1}{2}\times\frac{5}{48}t^2$; $t=134\pm101\cdot3\ldots$

IX.

1. 10 velos; 2 celos; velocity at beginning $=6$ velos; velocity at end $=14$ velos. **2.** 80 velos; 20 seconds previously; 800 ft. back.

3. $6\frac{2}{3}$ celos. **4.** 2 celos. **5.** 10 sec. **6.** 10 sec.

7. In $2\frac{1}{2}$ sec. and 5 sec. **8.** 20 celos. **10.** 4 sec.

12. 48 ft. from A. **13.** 90 feet from B.

14. In $\sqrt{\dfrac{a}{2g}}$ sec. at a point $\frac{1}{4}a$ feet from A. **16.** $2\frac{1}{2}$ secs.

X.

1. $\frac{1}{2}$ celo. **2.** 32 celos. **3.** $\frac{1}{2}$ celo. **4.** $12\frac{4}{5}$ celos.
5. $\frac{3}{7}$ celo. **6.** $\frac{1}{64}$ celo. **7.** 5 velos; 25 ft.
8. 48 velos; 96 ft. **9.** 2 velos; 3 ft. **10.** $5\frac{1}{7}$ velos; $23\frac{1}{7}$ ft.
11. 3 velos; 18 ft. **12.** 16 velos; 4 ft.
13. (i) 8 velos; 18 ft. (ii) 22 velos; 84 ft. (iii) 158 velos; $283\frac{1}{2}$ ft.
14. 120 ft. **15.** 256 ft. and back again.
16. after 2 sec.; 32 velos; or, after 4 sec.; 32 velos.
17. after 2 sec.; and after 3 sec. **18.** 54 poundals.
19. 32 poundals; 36 velos. **20.** 36 velos; 320 poundals.
21. 20 lbs. **22.** $\frac{40}{11}$ lbs. **23.** $14\frac{37}{42}$ ft.
24. One is four times as great as the other. **25.** $\frac{60np}{m}$ velos.
26. 180 poundals. **27.** About 4 m. 10 secs.; about 5·3 velos.

XI.

1. mg poundals. **2.** $\frac{ft^2}{40 \times 112}$ feet. **3.** $20 \times 112\, kg$.
4. $\frac{4928}{1125}\, mk$ secs.

XII.

1. 9 pound-velos. **2.** 1120 pound-velos.
3. $100 \times 20 \times 112 \times 44$ pound-velos. **4.** $62\frac{1}{2}$ pound-velos.
5. 336000 pound-velos. **6.** $2956\frac{4}{5}$ pound-velos.
7. 80 pound-celos. **8.** 224×32 pound-celos.
9. $\frac{44 \times 5 \times 20 \times 112}{3}$ pound-celos. **10.** $10416\frac{2}{3}$ pound-celos.
11. $56 \times \frac{1}{20} \times (1500)^2$ pound-celos. **12.** $\frac{88 \times 88}{2700} \times 12 \times 14$ pound-celos.

XIII.

1. (7) 80 poundals. (10) $10416\frac{2}{3}$ poundals.
(8) 32×224 poundals. (11) $\frac{56}{20} \times (1500)^2$ poundals.
(9) $\frac{44}{60} \times 100 \times 20 \times 112$ p. (12) $(\frac{88}{3})^2 \times \frac{1}{300} \times 168$ poundals.

2. (1) $\frac{9}{20}$ poundals. (4) $3\frac{1}{8}$ poundals.
(2) 56 poundals. (5) 16800 poundals.
(3) 492800 poundals. (6) $147\frac{21}{25}$ poundals.

3. $16\frac{32}{75}$ minutes. **4.** 18000 poundals. **5.** 80 poundals.
6. $36\frac{68}{135}$ poundals.

XIV.

1. 4 sec. **2.** (i) 48 ft. (ii) 112 ft. **3.** 10 sec.; 1600 ft.
4. 64 velos; 64 ft. **5.** In 4 sec. from time stone is thrown.
6. 2 sec.; 192 ft. from the foot of the tower. **7.** 2 celos.
8. $\frac{4 \times 100 \times 112}{3^2}$ lbs. weight. **9.** 480 velos.
10. $(480)^2 \left\{\frac{1}{7680} - \frac{1}{11340}\right\}$ ft. $= 4{\cdot}8\ldots$ ft. **11.** $\frac{28}{15}$ lbs. weight.
12. 14m. 40 sec. **13.** In 50 secs. **14.** 308 secs.

XV.

1. $\frac{16}{100} = \frac{4}{25}$ celos. **3.** Their ratio is 140 to 147. **4.** $\frac{16}{21}$ ft.
5. $\frac{64}{121}$ ft. **6.** 4 × 14 × 22 poundals. **7.** 13 sec.
8. 770 lbs. weight. **9.** 17570 lbs. weight.

XVI.

1. 300 × 20 × 112 poundals.
2. 250 × 20 × 112 poundals; 100 × 20 × 112 poundals;
200 × 20 × 112 poundals; 50 × 20 × 112 poundals.
150 × 20 × 112 poundals;
3. 500 × 20 × 112 poundals.
4. (i) $\frac{220 \times 20 \times 112}{3}$ poundals. (iii) $\frac{22 \times 20 \times 112}{3}$ poundals.
(ii) 44 × 20 × 112 poundals. (iv) 176 yards.
5. (i) $\frac{8a}{15}$ celos. (ii) $\frac{7ma}{15}$ poundals. (iii) $\frac{ma}{5}$ p. (iv) $\frac{ma}{15}$ p.
6. (i) $\frac{10a}{10+n}$ celos. (ii) $\frac{n-4}{10+n}p$ poundals.

XVII.

1. $\frac{256}{3}$ poundals; 2 celos. **2.** 120 poundals; 8 celos.
3. $\frac{576}{12}$ poundals; $\frac{4}{3}$ celos. **4.** $\frac{256}{2}$ poundals; 2 celos.
6. $\frac{1}{2}$ a second. **7.** $30\frac{35}{66}$ ozs. **8.** $15\frac{1}{2}$ ozs. **12.** $\frac{f-f'}{m+m'}$, poundals.
14. Velocity before : velocity after $= \sqrt{3} : \sqrt{2}$.
15. 22 × 112 poundals. **16.** The weight of $1\frac{1}{2}$ cwt. **18.** 3600 ft.
19. $\frac{63}{64}$ of the weight of 2 ozs. **20.** $\frac{127}{128}$ of the weight of 2 ozs.

XVIII.

1. 896 × 2000 pulses; the weight of 1250 tons. **2.** $\frac{1}{50}$ sec.
3. 1600 × 1800 pulses; 803$\frac{4}{7}$ tons weight. **4.** $\frac{1}{20}$ sec.
5. 32 pulses; 10 lbs. weight. **6.** 24 pulses.
7. 66 × 20 × 112 pulses; weight of 2$\frac{1}{16}$ tons.
8. 55 pulses; 17$\frac{3}{16}$ lbs. weight.
9. (i) a weight of 160 tons. (ii) 3 in.
10. Force = weight of 10$\frac{5}{7}$ cwt.; interval = $\frac{1}{480}$ second.
11. Force = weight of 8 tons; interval = $\frac{1}{16}$ second.

XIX.

1. 10 velos. **2.** $\frac{mv}{m+m'}$ velos. **3.** 9 velos. **4.** $\frac{8}{15}$ velos.
5. 5$\frac{1}{2}$ velos. **6.** 22$\frac{1}{2}$ velos. **9.** 5 velos. **10.** $\frac{44}{9}$ velos.
11. $\frac{1200}{17921}$ velos. **12.** $(\sqrt{6}+\sqrt{2})$ sec. = 3·8636 sec.

XX.

1. 18$\frac{1}{5}$ velos; 22$\frac{1}{5}$ velos. **2.** $-2\frac{2}{3}$ velos; 5$\frac{2}{3}$ velos.
3. $-\frac{1}{2}v$; $5\frac{1}{2}v$. **4.** 4 velos.
6. $\left(\frac{e+1}{2}\right)v$; $\left(\frac{e+1}{2}\right)^2 v$; $\left(\frac{e+1}{2}\right)^3 v$; $\left(\frac{e+1}{2}\right)^n v$.
7. In 2 secs.; 16ft. below the point from which m' is let fall;

$$v = -\frac{32}{m+m'}\{m'+2em'-m\}; \quad v' = \frac{32}{m+m'}\{m+2em-m'\}.$$

8. $\frac{1}{1+2e}$. **9.** 7$\frac{1}{2}$ velos in opposite direction to shot.

11. The impulsive tension causes a stress of opposite *sign* to that caused by an impact, hence we have $v+v'=u$ and $v-v'=eu$, whence $v=\frac{1}{2}u(1-e)$; $v'=\frac{1}{2}u(1+e)$ after the first impact, the velocities are $\frac{1}{2}u(1+ee')$; $\frac{1}{2}u(1-ee')$. **12.** 2 velos; $\frac{4}{3}\sqrt{3}$ velos; $\frac{1}{4}\sqrt{13}$ velos.

XXI.

1. 1 second. **2.** $\sqrt{\frac{2}{3}}$.
3. (i) e^2h. (ii) $\frac{2e\sqrt{2h}}{\sqrt{g}}$. (iii) $2\left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h}{g}}$.
4. $\frac{2h}{1-e^2}-h$. **6.** 15·34... velos.

XXII.

1. $\frac{1}{20}$ velo; $\frac{1}{1200}$ celo. **2.** $\frac{22}{15}$ velo; $\frac{11}{27000}$ celo.

3. $\frac{3a}{b}$ velos; $\frac{3a}{b^2}$ celos. **4.** $\frac{3}{88}$ sec. **5.** 25 ft.

6. $32\sqrt{330}$ velos. **7.** $\frac{16000}{11}$ sec.; $(\frac{40}{11})^2 \times 400$ feet. **8.** 50 ft.

9. 32 lbs. **10.** $\frac{20 \times 112 \times 5280}{(60)^4}$ poundals. **11.** $\frac{32}{3 \cdot 28}$.

12. $\frac{fn}{m^2}$ celos.

14. (i) ·0328... velos. (ii) ·0328 celos.
(iii) (·0328...) × (·0022...) poundals.

XXIII.

1. 500 yards. **2.** 5 miles. **3.** 10·02 ft. nearly. **4.** 25 yards.
5. 103·6 ft. **6.** 14 feet; 2 feet; 10 feet; 13 feet nearly.
7. 350 yards; 60^0 with first direction.
8. 115·6 yds.; making an angle $\tan^{-1} \frac{\sqrt{2}}{10+\sqrt{2}}$ East of North.
9. $20\sqrt{3}$ yards.
10. 3·6 miles, in a direction making the angle $\tan^{-1} \frac{1}{6}$ with the North.
11. 4·836 miles; $\cos^{-1} \cdot 874753 = 28^0\ 59'\ 7''$ [by the Tables].
12. $\frac{1}{6}\sqrt{\{(36)^2 + (23)^2\}}$ miles.

XXIV.

1. 50 yards; 86·5... yards. **2.** 15 yards, 20 yards.
3. 323·8 yards. **4.** $\sqrt{\{(662 \cdot 13)^2 + (212 \cdot 13)^2\}}$.
5. ·707... mile; 0 miles. **6.** 180 yards.

XXV.

1. 150 feet from its initial position; in a direction making an angle whose sine is $\frac{4}{5}$ to the East of North.
2. $\sqrt{\{(30)^2 + (45)^2\}}$ feet from its initial position.
3. 24 ft. south of its initial position.
4. $\sqrt{\{(35)^2 + (15)^2\}}$ feet from its initial position.
5. $\sqrt{\{(200)^2 + (30)^2\}}$ ft. from its initial position.
6. The point is at rest.
7. $\sqrt{\{(40)^2 + (64)^2\}}$ ft. from its initial position.
8. 20 ft. to the north of its initial position.

XXVI.

	Range.	Time of flight.	Greatest Height.
1.	312·5 feet.	4·42... secs.	78·125 feet.
2.	78·125 feet.	2·21... secs.	19·53125 feet.
3.	128 feet.	2·828... secs.	32 feet.
4.	1691·4 feet.	7·8 secs.	244 feet.
5.	270·6 feet.	5·4 secs.	117·2 feet.
6.	67·6 feet.	2·7 secs.	29·3 feet.
7.	168·75 feet.	2·88 secs.	126·5625 feet.
8.	468·75 feet.	$6\frac{1}{4}$ secs.	156·25 feet.

9. 625 feet nearly. **10.** 300 yards nearly. **11.** $1177\frac{1}{2}$ feet.
12. 19 inches and 1·88 feet. **13.** 9114 yards. **14.** 18454 yards.

XXVII.

1. 20 velos. **2.** 25 velos. **3.** 50 miles per hour $= 73\frac{1}{3}$ velos. **4.** 8·2...velos.
5. $\sqrt{a^2+b^2}$ velos. **6.** $\sqrt{a^2+\left(\frac{22m}{15}\right)^2}$ velos. **7.** 3 velos.
8. $a\sqrt{3}$ velos. **9.** 6·7...velos. **10.** $10\sqrt{7} = 26·4...$velos.
11. 20 velos. **12.** 42·7 velos. **13.** 57·46 ft. see Ex. i. p. 72.
14. 35·35...velos, 35·35...velos. **15.** 99·6 velos.
16. 1136 velos nearly. **17.** 28·28...velos. **18.** 120^0 from N.
19. It has received $(2-\sqrt{2})$ times its former velocity, making an angle $\cos^{-1}\frac{1}{2}(2-\sqrt{2})$ with it. **20.** 67 velos nearly. **21.** 30 velos.
22. 20·5 velos making angle with horizon $\tan = \frac{9}{5}$.
23. At a point $\sqrt{(20)^2+(\frac{88}{3})^2}$ ft. from point of stoppage.
24. 21 velos nearly. **25.** $8\sqrt{2}$ miles per hour.

XXVIII.

1. Angle in direction of train whose $\tan = -\frac{3}{4}$. **2.** 13 celos.
3. 13 velos. **4.** 50·4...velos. **5.** 45·5 velos nearly.
6. 135^0 with direction of train.

XXIX.

1. 1 celo; 6 secs. **2.** 4·4 secs. **3.** $2\sqrt{2}$ secs.; 96 feet.
4. 8 celos; $\tan^{-1}\frac{3}{4}$. **5.** 4·8 celos. **7.** $\frac{1}{4}\times 5·17... = 1·29...$celos.
8. 5·746...celos [see p. 73]. **11.** 2 : 1.

XXX.

2. $10\sqrt{(660)}$ seconds. **3.** $32\sqrt{(\frac{33}{10})}$ seconds.

4. The number of seconds t_1 is the positive root of the equation

$$h = 0\sqrt{(2g\sin\alpha l)}\sin\alpha t + \tfrac{1}{2}gt^2,$$

the horizontal distance $=\sqrt{(2g\sin\alpha l)}\cos\alpha t_1$ ft. **6.** 30^0.

7. Through the fixed point draw a horizontal straight line; describe a circle to touch this line at the fixed point and also to touch the fixed line. The line joining the two points of contact is the required direction.

9. The construction is the same as that in Example **7.**

XXXI.

1. After $\frac{15}{32}$ seconds.

3. (i) After $12 \pm \sqrt{119}$ seconds, (ii) at an angle with the horizon whose tangent is $\pm\frac{\sqrt{119}}{5}$.

XXXII.

1. The axis of the parabola is the vertical line passing through the point at which the ball was dropped. The directrix is the horizontal line 121 feet above this point. **2.** 221 ft. 8 in. with $128 \times \sqrt{3}$ velos.

4. With $20\sqrt{2}$ velos.

5. $\tan\alpha$ is one of the roots of the equation $64\tan^2\alpha - 25\tan\alpha + 26 = 0$.

6. The least velocity is $u\cos\alpha$ which it attains after $\frac{u\sin\alpha}{g}$ seconds at a point whose vertical distance is $\frac{u^2\sin^2\alpha}{2g}$ ft. and whose horizontal distance is $\frac{u^2\sin 2\alpha}{2g}$ ft. **10.** 15^0. **12.** $\frac{1}{2}$ second.

13. $12\sqrt{3}$ feet.

14. The time t_1 is the positive root of the equation

$$gt^2 + 2\sqrt{(2ga\sin\alpha)}\sin\alpha t = 2h,$$

and the distance $BC = \sqrt{\{h^2 + 2ga\sin\alpha t_1{}^2\cos^2\alpha\}}$ ft.

15. 22 feet. **16.** $\frac{1}{2}\sqrt{1189}$ ft. **17.** $\frac{1}{4}\sqrt{15313}$ velos.

18. $u\cos\alpha$ velos. **19.** $u\cos\alpha\sec\beta$ velos. **21.** 1 foot.

XXXIII.

1. $(1+e)$ 10 velos.

2. They must interchange velocities in the line of impact and therefore [Ex. ii. p. 59] they must be of equal mass and perfectly elastic. **3.** Their velocities are equal and each $=u\sqrt{\{\frac{1}{2}(1+e^2)\}}$.

4. $\sqrt{\{u^2+\frac{1}{4}(1+e)^2u'^2\}}$ and $\frac{1}{2}(1-e)u'$.

5. (i) 100 velos. (ii) 5 seconds.

6. (i) 50 velos. (ii) 2·8... seconds [Ex. iii. p. 36].

XXXIV.

1. 32 feet. **2.** 1080 feet.

3. $u\sqrt{[\{\cos\theta\sec\alpha-\sin(\theta-\alpha)\tan\alpha\}^2+e^2\sin^2(\theta-\alpha)]}$.

6. Let the angle of projection with the horizon be α, then the line of flight from the horizontal motion is $\frac{a}{u\cos\alpha}+\frac{2a}{eu\cos\alpha}+\frac{a}{e^2u\cos\alpha}$ and from the vertical motion the time is $\frac{2u\sin\alpha}{g}$. Therefore

$$u=\frac{g}{\sin 2\alpha}\left\{1+\frac{2}{e}+\frac{1}{e^2}\right\}.$$

7. $\cot\alpha=2$.

9. The path of the ball is always parallel to a diagonal and it always returns to the point of projection. **13.** $2eh\sin 2\alpha$ feet.

14. $\sin 2a=\frac{ag}{eu^2}$.

15. [See Ex. **6.**] From the horizontal motion the time of flight $t_1=\frac{b}{u\cos a}\left\{1+\frac{2}{e}+\frac{1}{e^2}\right\}$. The time t_2 of going up to the ceiling is the least root of the equation $h=u\sin\alpha t-\frac{1}{2}gt^2$. The time of coming down t_3 is the positive root of the equation $h=evt+\frac{1}{2}gt^2$, where

$$v^2=u^2\sin^2\alpha-2gh;\ \text{also}\ t_1=t_2+t_3.$$

XXXV.

1. 20 celos. **2.** 40 poundals = weight of $1\frac{1}{4}$ lbs.

3. $\frac{mv^2}{p}$ feet. **4.** The weight of $1026\frac{2}{3}$ lbs.

7. The weight of $22\frac{44}{75}$ lbs.

8. (i) When at rest the tension is 56 lbs. weight.

(ii) When in motion the tension is (56 + 70) lbs. weight.

9. The difference is about $94\frac{1}{2}$ lbs. weight.

10. At the distance of 3 feet from the 4 lbs.

11. 8 velos. **12.** $\frac{50}{32}$ lbs.

XXXVIII.

1. $40 \times 56 \times g$ foot-poundals; 2240 foot-pounds.

2. $10 \times 14 \times 500 \times g$ foot-poundals; 70000 foot-pounds.

3. $10 \times 2240 \times 500 \times g$ foot-poundals; 11200000 foot-pounds.

4. $-12 \times 14 \times 200 \times g$ foot-poundals; -33600 foot-pounds.

5. $300 \times 2240 \times \frac{1}{50} \times 1760 \times 3 \times g$ foot-poundals; 433843200 foot-pounds.

6. $224 \times 600 \times g$ foot-poundals; 134400 foot-pounds.

7. 9 : 880. **8.** 22400 foot-pounds. **9.** $234\frac{2}{3}$ foot-pounds.

10. 50 velos $= 34\frac{1}{11}$ miles per hour. **11.** $224 \times 8 \times 88$ foot-pounds.

12. $19\frac{1}{5}$ miles per hour.

13. $\dfrac{16 \times 60}{24 + 10 \cos a}$ miles per hour, where $\cos a = \sqrt{\{1 - (\frac{1}{50})^2\}} = {\cdot}9998\ldots$ that is, $19\frac{1}{5}$ miles per hour, very nearly. **14.** 29568 foot-pounds.

15. 2 : 5.

XXXIX.

1. 800 foot-poundals. **2.** $(2000)^2 \times 4 \times 112$ foot-poundals.

3. $(1600)^2 \times 6 \times 112$ foot-poundals.

4. $(88)^2 \times 200 \times 2240$ foot-poundals.

5. $\frac{1}{25} \times (88)^2 \times 84$ foot-poundals $= 813{\cdot}12$ foot-pounds.

6. 1680 foot-pounds.

7. $\{(66)^2 \times 180 \times 2240 + 3 \times 1120 \times 1760 \times 6\}$ foot-poundals

$$= 2240 \times 9 \times 11 \{7920 + 1760\} \text{ foot-poundals.}$$

8. $\{(88)^2 \times 50 \times 2240 + 500 \times 100 \times 2240 \times 32\}$ foot-poundals

$$= 2240 \times 64 \times 50 \{121 + 500\} \text{ foot-poundals.}$$

9. The necessary work is $112 \times 8 \times 32$ foot-poundals; the superfluous work is $112 \times \frac{1}{2} \times 625$ foot-poundals. Their ratio is 512 : 625.

10. No work is done while the brakes are applied, that is, for 110 yards. The work done in producing kinetic energy is $\frac{1}{2}(11)^2 \times 2240 \times 10$ foot-poundals, the work done against friction is $28 \times 32 \times 1650 \times 3$ foot-poundals. The total work done is

$$2240 \times 11 \{55 + 180\} = 2240 \times 11 \times 235 \text{ foot-poundals.}$$

The work done in going a mile with uniform velocity

$$= 28 \times 32 \times 1760 \times 3 = 2240 \times 11 \times 192 \text{ foot-poundals.}$$

These two are in the proportion 235 : 192.

11. Let the train be m lbs. The work done in the first case is $\{\frac{1}{2}(66)^2 m+\frac{32}{120}\times m\times 1760\times 3\}$ foot-poundals, in the second

$$\tfrac{32}{120}\times m\times 1760\times 3 \text{ foot-poundals.}$$

Their ratio is 3586 : 1408.

12. $11\times 2240\{55-4\}$ foot-poundals.

XLI.

2. 80 velos.

7. See Art. 133.

14. $\dfrac{(2000)^2}{(2000)^2-(1800)^2}=5\tfrac{5}{19}$ planks.

19. Let the rod of the pendulum make the angle θ with the vertical, then $\theta=60^0$.

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